Some Challenges for the Improvement of Doubling at a Virtual Racecourse

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Abstract

In this paper, I try to generalize the method of doubling (the rule of doubles) such that rewards can be obtained with lower risk. Since it is said that applying doubling generally leads to bankruptcy, I introduce an expansion method so that the risk is reduced. That is, instead of doubling the bet after a loss, we wait to increase the bet until the uncollected amount reaches a predetermined value. I enter to compete in a single win of horse racing at a virtual racecourse by betting the same rank or the same horse number rather than predicting. Further, I propose some improved models, skip low odds races, skip all races after a win until the next day, and start betting with a low-odds first-rank horse. Next, I evaluate models for how much of an input amount is needed to be risked. Then, I show the results generated by 100,000 races. It is discovered that the input amount may be increased 10,000 times, which may dissuade many people from using this method. Finally, I propose some ideas for future study, including betting methods for multiple horses, and for one-horse multiple-lines methods. While risk cannot be completely eliminated, we can observe upward trending except for some deep valleys in profit graphs. Horse racing is a special type of gambling

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whose past data is publicly available. This includes data such as hit rate, recovery rate, and average refunds by rank or horse number. I believe that this study can become a foothold to develop a winning method in the future.

Keywords: Doubling. Martingale. Horse racing. Recovery rate. Hit rate.

Section 1 Introduction

In this paper, I attempt to make a profit with low risk by using an extension of the method of doubling. We are able to find data in Japan where the recovery rate is 80% or over in single-win betting for predictions of first place. So, my goal is to increase the recovery rate to over 100% with low risk by betting the same rank or same horse number rather than using predictions.

Doubling is well-known as a dubious winning method. The strategy is simple: Keep doubling the bet after losing until a win is achieved. If we have enough funds and the return, or payout, is over 200%, then a profit over 100% is guaranteed. Turner (1998) compares doubling to constant betting by using computer simulations. Applying this to horse racing is widely discussed. We can find many websites and movies about applying doubling to horse racing through search engines by using the keywords “Martingale methods” and “horse racing”. A simple application of this method is as follows: Bet on a horse whose odds is around 2, then double the bet after losing. Some websites propose alterations or modifications. Umameshi.com proposes some alterations of doubling that are focused on odds, for example. Similar to the standard doubling method, these alterations also carry the risk of insufficient funds.

Here, I discuss an expansion of this method which suppresses the input amount (the wager) to our tolerable range and reduces the risk of bankruptcy. The method is as follows: We predetermine the value that we will wait to reach in order to increase the bet. We compute the input amount using this value and the uncollected amount (the temporary accumulated loss). After winning such that uncollected amount is zero, we set the input amount to the initial amount. By repeating this, we expect to see improvement of the recovery rate with a small amount of funds. As a risk evaluation index, we use the maximum input amount in 10,000 races.

In Section 2, I explain the simulation data used in the following sections. Section 2 is not related to the methods discussed in the following sections and can be skipped if there is no interest in the data creation process.

In Section 3, I define constants and variables, and I formulate the standard model.

In Section 4, I show examples of two strategies. One strategy is betting on horses of the same rank, and the other is betting on the same horse number. Predictions are not used. In Japan, horse numbers are positioned from the inside. I show how the profit changes in both good and bad situations. Further, I discuss the effects of changing parameter values.

In Sections 5 through 7, I propose some improved methods. I design several models that aim to reduce the maximum input amount to 100,000 yen or less over 10,000 races. There are two reasons why I set the upper limit to 100,000 yen. One is based on my experience of using doubling with an initial bet of 100 yen. After a long losing streak, it becomes mentally difficult to continue to use this strategy. Confidence is lost in one’s own betting, which is not necessarily due to their remaining account balance. The other reason is the possibility of making the odds lowered and the returns reduced due to one’s own invested bets. In Japan, aside from graded races, it seems like that there are not many large bets over 100,000 yen. (I don’t have accurate information about this.)

In Section 5, using fixed parameters and a predetermined rank or horse number, some improvements are proposed such that the input amount is lowered by a reduction of betting frequency. That is, under certain conditions we skip some races, skip low-odds races when an uncollected amount remains, skip all remaining races after a win such that the uncollected amount becomes zero, and start betting at the predetermined low-odds, most popular horse whose probability of winning is high.

In Section 6, for reduction of risk in the case that the uncollected amount becomes large, I propose increasing the parameters so that we can store a higher uncollected amount.

Further, in Section 7, I propose a method for betting on the lower-ranked horses which can store larger uncollected amounts as a workaround for the case where the uncollected amount becomes large.

In Section 8, I show the results of 100,000 races using the parameters and models constructed in previous sections. In actuality, I constructed the models such that the input amount does not exceed 100,000 yen. I confirmed whether the input amount exceeded 100,000 yen in 100,000 races in order to look ahead to actual operation in practical use. There were a few cases where the input amount exceeded 100,000 yen. Furthermore, in 100,000 races, there were also some cases where the input amount grew to 10,000 times the initial amount. Due to the fear of doubling, we cannot say that it is safe to use the model in 10,000 races, and we cannot determine how much preparation would be necessary for actual operation in practical use. This is what I want to emphasize most in this paper.

In Sections 9 and 10, I describe some ideas for future study. In Section 9, I try to recover from a large uncollected amount. In Section 10, I introduce a multiple-line model. For the case of betting on multiple horses in the same race, I show how to bet on one horse divided into multiple lines to reduce the increase of the input amount. In the multiple-line model, we can reduce risk by clearing the uncollected amount when the total profit is updated. Moreover, it is effective to use the multiple-line model in the case where the uncollected amount becomes large in the one-line model.

Finally, in Section 11, I conclude this paper. In addition, although it may be superfluous, I write about my dreams and delusions. While there is a risk that the input amount grows to over 100,000 yen, we can confirm that graphs show that profits are generally increasing. There are some techniques which lead to reducing this risk. I hope this paper will serve as a stepping stone for the development of a reliable method of winning.

Gerolamo Cardano said that the biggest advantage in gambling is not to gamble at all: “*Il vantaggio più grande del gioco d'azzardo sta nel non giocare affatto.*” I aim to challenge this quotation.

Section 2 Constructing horse racing data

Before the discussion, I will explain how to construct the racing data and the results. For verifying some of the models, it is important to use actual data, needless to say. However, I did not obtain permission to use the racing data that originates from organizations or tipsters. Further, even if I had access to that data, it is difficult to gather a sufficient amount to be able to evaluate the results.

For this paper, I constructed racing data for 110,000 races. The data from the first 10,000 races was used to construct the models, and the data from the next 100,000 races was used to confirm the amount of risk that these models contain.

In actual races, there are many factors, including race course conditions and the weather. There are also racing types such as handicap/non-handicap racing and different racing classes. However, in this paper, we will construct data for just one type of race.

Variables described here are only used in this section. This is unrelated to the expansion methods of doubling, and you can skip this section if you are only interested in those methods.

2.1 The racing model

I constructed the racing data by making simple assumptions. The number of horses for each race varies from 6 to 16, and this number will be decided by a roulette wheel with predetermined arc lengths. Uniform random variables will decide how many horses run in each race. The strength of each horse is decided by an exponential distribution. The odds for each horse are decided by two votes based on voting behaviors. One vote is dependent on strength, and the other is independent of strength. Each horse number has an advantage/disadvantage. Returns can change slightly with a normal distribution. The winner is decided by a roulette wheel where the arc length for each horse is determined by its strength and horse number.

The processes are as follows:

PROCESS1: Setting the number of horses, N_H

I set the number of horses for each race by using uniform random variables (the RAND function in Microsoft Excel). Let u be a uniform random variable from 0 to 1. Using Table 1, for example, if u is 0 to 0.01, the number of horses is 6, and if u is 0.01 to 0.03 (=0.01+0.02), the number of horses is 7.

Table 1. Occurrence probability for the number of horses

Number of horses	Occurrence Probability(%)	Number of horses	Occurrence Probability(%)	Number of horses	Occurrence Probability(%)
6	1	10	7	14	11
7	2	11	8	15	14
8	3	12	9	16	30
9	5	13	10	Sum	100

PROCESS2: Setting the strength of each horse i, s_i

Strength is set by using exponential random variables. For each horse, let u_1, u_2, u_3 be uniform random variables from 0 to 1. The strength is set to:

$$(1)\qquad s_i = \begin{cases} \text{If } u_1<\alpha_A, & \min(\alpha_B - \varepsilon\times u_2,\ \alpha_C - \alpha_D\times\log u_3) \\ \text{Else} & \min(\alpha_B - \varepsilon\times u_2,\ \alpha_C - \alpha_E\times\log u_3) \end{cases}$$

$$(\alpha_A{:=}0.3,\ \alpha_B{:=}400,\ \alpha_C{:=}1,\ \alpha_D{:=}330,\ \alpha_E{:=}50,\ \varepsilon{:=}0.001)$$

The ratio is set to $p_i=s_i\div\Sigma_j s_j$. Furthermore, this also sets the rank of each horse, r_i. That is, if p_i is the largest value, then r_i=1, and p_i is the smallest value, then $r_i=N_H$ for the race.

PROCESS3: Setting the rough sum vote related to strength, N_S

I am using exponential random variables. For each race, let u be a random variable from 0 to 1. The vote total (the number of bets) is set to 500,000-1,000,000× log u.

PROCESS4: Setting the rough sum vote not related to strength, N_E

It is set to $N_S \div \alpha_D$ (α_D:=50).

PROCESS5: Setting the vote-related strength of each horse i, v_i

We can assume that the number of votes is the sum of two numbers.

The first vote follows a binomial distribution with the parameters rough sum total, N_S, and each horse's strength ratio, p_i. This assumption is based on those voters (bettors) whose forecast is formed from an analysis of horse strength.

The second vote is set to $N_E \div N_H$. This assumption can be explained by the fact that some people do not forecast and are motivated to buy tickets based on the amount of cheering, the horse name, the jockey, the horse number, and so on.

Moreover, I use an approximation of normal distribution using uniform random variables and an approximation of binomial distribution using normal distribution (see Box and Muller, 1958). That is, u_1, u_2 are set to uniform random variables from 0 to 1, and we can compute the number of votes as follows:

(2) $v_i^{(S)} = \{(-2\times\log u_1)^{0.5}\}\times\cos(2\times\pi\times u_2)\ \{N_S\times p_i\times(1-p_i)\}^{0.5} + N_S\times p_i$

(3) $v_i^{(E)} = \{(-2\times\log u_1)^{0.5}\}\times\sin(2\times\pi\times u_2)\ \{N_E\div N_H\times(1-(1\div N_H))\}^{0.5} + N_E\times(1\div N_H)$

The total votes for horse i, v_i is expressed as:

(4) $v_i = v_i^{(S)} + v_i^{(E)}$

As a side note, $\Sigma v_i^{(S)} \neq N_S$, $\Sigma v_i^{(R)} \neq N_R$. Even if $r_i > r_j$, it may be the case that $v_{ri} < v_{rj}$.

PROCESS6: Setting the odds for each horse, o_i

Since the refund (or payout) rate for winning tickets is 80% in Japan, o_i is set as follows:

(5) $o_i = \lceil 8 \times v_i \div \Sigma v_j \rceil \times 10$

where $\lceil x \rceil$ represents the round-up value of x.

PROCESS7: Setting the advantage/disadvantage for each horse number, w_i

Looking at actual racing data, we can see that there is an advantage/disadvantage by horse number for each race course. Accordingly, we will set an advantage/disadvantage for each horse number, w_i , using the horse number advantage coefficient, c_i , as follows (see Table 2):

(6) $w_i = s_i \times c_i$

Table 2. Advantage/disadvantage coefficient for each horse number

c_i	i	c_i	i	c_i	i	c_i
0.97	5	1.07	9	0.92	13	0.9
1.05	6	1.04	10	1.07	14	0.92
1	7	1.08	11	0.97	15	0.93
1.12	8	1.07	12	1.05	16	0.95

PROCESS8: Deciding on the winning horse and setting the return

I am using uniform random variables to decide the winning horse number. That is, horse i will win with a probability of $w_i \div \Sigma_j w_j$. In general, the odds change after tickets are purchased, so I assume that the return for horse $i*$, r_{i*} is changed by a normal distribution of deviation, $o_{i*}÷100$. Furthermore, return amounts are in units of 10 yen in Japan, so the return is set as follows:

$$(7)\quad r_{i*}= o_{i*}+o_{i*}÷100×\{(—2×\log u_1)^{0.5}\}×\sin(2×\pi×u_2)\ \{ o_{i*}÷100\}×10$$

PROCESS9: Delete abnormal data

Finally, the following three cases involving abnormal data are deleted because we rarely observe them in actual races:

・Odds of the first most popular horse are 1.0 or under.

・Odds of the second most popular horse are 2 or under.

・Returns are 100 yen or lower.

2.2 The results of 110,000 races

To begin, I constructed data for 10,000 races using the processes of the previous section. The racing data that were deleted during PROCESS9 totaled 41 results.

Tables 3 to 12 show the results of racing data for 10,000 races. We will use these data in Sections 3 to 8 and refer to them as data0.

Table 3 shows the odds in units of 100 yen. Percentage points are computed by the PERCENTILE function in Microsoft Excel.

Table 4 shows the results of returns by rank. It can be seen that the recovery rates of the high-rank horses are close to the refund rate of 80%. This phenomenon can also be observed in actual racing data.

Table 5 shows the results of returns by horse number. Compared with Table 2, the advantage coefficient affects some numbers' recovery rates, while others are not affected.

Table 6 shows the results of the most popular horse whose odds are 2 or lower. It can be seen that hit rates are high while recovery rates are relatively low.

Tables 7 and 8 show the change of hit/recovery rates. When the number of races is large, the change is small. This may be interpreted as the Law of Large Numbers. There is a risk of overestimation or underestimation if we use too little historical data that covers a short period of time.

Tables 9 and 10 show the results by the number of horses.

Tables 11 and 12 show losing streaks and winning streaks. In these tables, 1 represents 1 loss or win.

I tried to ensure that the averages and standard deviations of odds and returns were made as close as possible to actual races. Equations (1) through (7) may be a bit complicated. While observing the results, I made adjustments to these equations and to the values of parameters. However, some indices can be close in value, yet others are not. Although there are different race types and conditions in actual races, I am only handling one situation here. It gets further complicated when considering actual processes and the behavior of voters. However, I am more interested in the topic of doubling, so let’s move ahead to that discussion.

Table 3. Results of odds at PROCESS6

Rank	min / max	avg / stdev	10% / 20%	Median	80% / 90%
1	110/ 700	265./ 84.	170/ 190	250	330/ 380
2	210/ 1640	463.9/ 158.2	300/ 340	430	570/ 660
3	290/ 3230	735./ 272.8	460/ 520	670	910/ 1090
4	400/ 6790	1061.7/ 423.3	660/ 740	960	1320/ 1590
5	530/ 10800	1457.6/ 640.	870/ 990	1310	1820/ 2190
6	630/ 16490	1947.7/ 1030.9	1100/ 1270	1710	2410/ 2960
7	750/ 20360	2541./ 1453.2	1373/ 1590	2190	3140/ 3970
8	870/ 21670	3287.4/ 2000.8	1670/ 1950	2750	4120/ 5380
9	1030/ 26470	4222.9/ 2709.9	2020/ 2390	3440	5380/ 7210
10	1110/ 28750	5275.5/ 3359.7	2440/ 2920	4290	6830/ 9310
11	1300/ 32060	6562.4/ 4146.1	2960/ 3540	5320	8724/ 11630
12	1630/ 33930	8089./ 4905.	3590/ 4360	6700	11000/ 14710
13	1720/ 38120	9968.8/ 5718.3	4420/ 5420	8450	13700/ 17960
14	2260/ 40400	12142.8/ 6229.1	5490/ 6870	10730	16778/ 21204
15	2670/ 42950	15610.7/ 7196.3	7330/ 9220	14330	21730/ 25976
16	3580/ 51070	20704.8/ 8250.9	10236/ 13130	19690	28098/ 32146

Table 4. Results or return by rank

Rank	min / max	avg / stdev	10% / 20%	Median	80% / 90%	hit / recovery rate	Prob. Under median
1	110/ 630	236.9/ 79.2	150/ 170	220	300/ 340	34.4/ 81.5	81.3～83.1
2	180/ 1280	410.3/ 135.8	270/ 300	390	490/ 590	19.4/ 79.7	90.2～90.7
3	260/ 2060	657.8/ 229.2	421/ 470	610	810/ 930	12.4/ 81.7	93.6～93.8
4	390/ 3510	934./ 352.1	590/ 670	850	1140/ 1370	8.7/ 80.9	95.6～95.7
5	500/ 3470	1252.4/ 428.6	810/ 900	1160	1548/ 1830	6.3/ 79.2	96.8～96.9
6	650/ 5890	1651.8/ 678.9	1010/ 1140	1490	2080/ 2504	5./ 82.4	97.5～97.6
7	810/ 7050	2044./ 812.	1254/ 1420	1890	2572/ 3182	3.7/ 75.3	98.2～98.2
8	960/ 9900	2666./ 1259.7	1490/ 1720	2320	3338/ 4192	2.8/ 73.8	98.6～98.6
9	1230/ 15710	3216./ 1695.9	1665/ 2012	2745	4154/ 5261	2.1/ 68.9	cannot compute
10	1510/ 13480	3983.9/ 2009.6	2196/ 2514	3530	5086/ 6402	2.1/ 82.7	99.～99.
11	1750/ 16480	4573.8/ 2475.4	2415/ 2766	4025	5760/ 7342	1.6/ 72.1	99.2～99.2
12	2170/ 19690	5673.3/ 3081.7	3090/ 3340	4680	7160/ 9750	1.3/ 73.4	99.4～99.4
13	2570/ 30630	8569.1/ 5906.1	3546/ 4432	6000	11500/ 16656	1./ 88.4	99.5～99.5
14	2800/ 25460	9022.7/ 4584.	3942/ 4910	8320	12354/ 14674	.7/ 61.	99.7～99.7
15	5360/ 21340	10516.4/ 4363.2	6231/ 7354	9365	13662/ 15543	.3/ 33.6	99.8～99.9
16	6290/ 38110	18571.5/ 9414.3	9088/ 11992	15590	26402/ 31502	.4/ 80.3	99.8～99.8

Table 5. Results of return by horse number

Horse Number	min / max	avg / stdev	10% / 20%	Median	80% / 90%	hit / recovery rate	Prob.Under median
1	110/ 25920	909.2/ 1636.9	180/ 230	420	1060/ 1989	7.7/ 70.2	99.6〜99.6
2	110/ 27210	971.4/ 1806.4	170/ 220	430	1240/ 2096	8.2/ 79.2	99.6〜99.6
3	110/ 15660	947.9/ 1615.7	170/ 230	420	1220/ 2050	7.5/ 71.2	99.6〜99.6
4	110/ 28630	1020.2/ 2149.4	180/ 230	420	1100/ 2076	8.5/ 86.2	99.6〜99.6
5	110/ 15210	986.8/ 1431.6	180/ 230	470	1360/ 2262	8.5/ 84.2	99.6〜99.6
6	110/ 16830	1034./ 1563.4	170/ 220	450	1400/ 2440	8.1/ 83.4	99.6〜99.6
7	110/ 32220	961.8/ 1857.6	175/ 230	450	1230/ 2035	7.9/ 76.3	99.6〜99.6
8	110/ 38110	905.1/ 1687.2	190/ 246	450	1260/ 2094	7.8/ 70.7	99.6〜99.6
9	120/ 13480	971.6/ 1347.1	180/ 230	460	1286/ 2328	6.7/ 65.3	99.7〜99.7
10	110/ 19550	1045.2/ 1819.5	190/ 250	490	1256/ 2370	7.4/ 77.1	99.6〜99.6
11	110/ 30630	1041.8/ 1917.6	190/ 230	460	1370/ 2300	7.4/ 77.2	99.6〜99.6
12	110/ 25460	1104.8/ 1812.6	210/ 260	480	1358/ 2764	7.2/ 79.7	99.6〜99.7
13	120/ 17920	1255.1/ 2130.	220/ 266	590	1514/ 2567	6.4/ 80.	99.7〜99.7
14	110/ 11610	1122./ 1575.2	210/ 250	510	1530/ 2770	5.7/ 63.8	99.7〜99.7
15	130/ 14570	1227.7/ 1755.1	230/ 276	505	1664/ 2948	6.2/ 76.7	99.7〜99.7
16	130/ 14280	1236./ 1998.2	200/ 250	560	1850/ 2655	5.9/ 72.3	99.7〜99.7

Table 6. Results of the most popular horse with low odds

Odds	number of races	hit /recov.		Odds	number of races	hit /recov.	Odds	number of races	hit /recov.
1.1	22	63.6 / 70.0		1.1	22	63.6 / 70.0	1.6	312	54.2 / 83.8
1.2	59	66.1 / 78.0		1.2	59	66.1 / 78.0	1.7	370	43.8 / 71.8
1.3	116	62.9 / 79.7		1.3	116	62.9 / 79.7	1.8	408	44.1 / 76.7
1.4	164	62.2 / 84.5		1.4	164	62.2 / 84.5	1.9	424	43.4 / 79.7
1.5	232	57.3 / 83.6		1.5	232	57.3 / 83.6	2.0	458	37.6 / 73.3

Table 7. Change of hit/recovery rates by rank

I_A, I_B: Ratio of the first period to the minimum of the second through the fifth periods.

Rank	1 to 100	101 to 200	201 to 300	301 to 400	401 to 500	I_A
1	30.0/71.2	39.0/84.3	27.0/68.5	38.0/88.6	27.0/63.4	89.0
2	21.0/85.2	19./73.1	18.0/75.5	17.0/65.8	22.0/85.3	77.2
3	15.0/94.5	9./52.2	17.0/107.8	11.0/71.6	9.0/66.0	55.2
4	9.0/77.5	13./135.	9.0/86.7	8.0/57.9	15.0/139.0	74.7
5	4.0/40.3	7./76.6	8.0/95.0	5.0/68.4	7.0/77.3	169.7
6	4.0/47.7	2./21.9	6.0/84.8	4.0/49.2	9.0/143.3	45.9
7	7.1/162.3	2./31.6	4.0/60.4	5.1/116.9	4.0/74.9	19.5
8	4.3/105.6	4./69.4	4.1/92.8	6.3/240.7	2.1/45.3	42.9
9	2.2/69.3	2.1/80.3	2.2/78.0	1.1/31.6	3.2/159.1	45.5
10	0.0/0.0	0.0/0.0	3.3/112.0	2.4/90.0	0.0/0.0	—
11	0.0/0.0	3.4/287.5	2.4/149.0	0.0/0.0	0.0/0.0	—
12	4.1/194.7	0.0/0.0	0.0/0.0	2.9/149.6	0.0/0.0	0.0
13	0.0/0.0	0.0/0.0	0.0/0.0	1.6/188.0	1.4/109.6	—
14	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	1.7/125.9	—
15	2.3/170.7	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0
16	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	—
Rank	1 to 2,000	2,001 to 4,000	4,001 to 6,000	6,001 to 8,000	8,001 to 10,000	I_B
1	33.9/77.1	35.7/86.1	34.1/81.7	34.2/81.4	34.3/81.3	105.5
2	19.3/79.2	18.9/78.2	20.1/83.0	19.1/77.6	19.9/80.6	98.0
3	12.1/82.4	11.6/72.9	12.2/79.8	12.8/81.7	13.6/91.6	88.5
4	9.7/86.9	8.3/81.	8.7/80.2	9.4/87.3	7.3/69.0	79.3
5	6.2/75.6	6.4/81.	6.3/76.8	6.7/85.9	6.1/76.5	101.2
6	4.8/76.1	5.5/96.8	4.7/76.3	4.8/75.5	5.3/87.4	99.3
7	3.7/79.1	3.6/78.9	3.9/80.1	3.5/66.6	3.6/71.6	84.2
8	3.7/99.3	2.3/61.9	2.3/66.	2.8/72.6	2.7/69.5	62.3
9	2.3/73.2	2./60.9	2.3/76.1	2.3/74.4	1.9/60.3	82.4
10	1.6/58.1	2.1/93.2	2.3/88.5	2./74.6	2.3/98.4	128.4
11	1.3/69.	2./83.4	1.8/88.	1.1/57.7	1.6/62.5	83.6
12	1.6/84.3	1.6/100.5	1.1/56.7	1.1/73.2	1.1/52.7	62.5
13	0.7/50.6	1.1/73.3	1.3/156.7	1.4/112.2	0.7/51.4	101.5
14	0.5/26.4	0.6/71.7	0.8/80.4	0.5/43.0	1.0/83.4	163.0
15	0.3/43.8	0.3/28.0	0.3/44.7	0.1/10.5	0.4/40.7	23.9
16	0.3/57.6	0.7/138.2	0.0/0.0	.5/99.0	0.7/107.3	0.0

Table 8. Change of hit/recovery rates by horse number

HorseNumber	1 to 100	101 to 200	201 to 300	301 to 400	401 to 500	I_A
1	6.0/60.5	7.0/135.1	7.0/50.0	4.0/109.9	5.0/55.6	82.6
2	11.0/144.3	8.0/47.9	7.0/28.1	7.0/113.6	11.0/84.0	19.5
3	6.0/35.8	7.0/63.9	9.0/167.6	10.0/71.9	8.0/74.4	178.5
4	9.0/107.6	8.0/182.7	9.0/50.9	7.0/46.8	8.0/43.2	40.1
5	14.0/181.4	7.0/36.6	12.0/148.4	7.0/92.0	7.0/54.4	20.2
6	11.0/91.8	8.0/80.0	9.0/56.6	11.0/113.7	5.0/24.5	26.7
7	7.1/46.5	8.1/54.9	5.0/30.6	12.1/72.3	6.1/109.2	65.9
8	5.4/17.5	5.1/13.8	11.3/111.4	6.3/23.6	13.4/94.4	79.0
9	4.5/42.8	7.3/44.3	4.3/19.8	3.3/61.4	5.3/47.4	46.2
10	3.5/15.9	14.1/81.1	7.8/105.1	11.8/172.8	11.4/80.3	504.3
11	6.3/31.6	5.7/32.0	6.0/72.1	6.3/52.3	4.8/82.0	101.3
12	6.8/132.3	9.3/77.2	3.8/45.8	8.6/131.1	7.9/137.4	34.6
13	7.4/55.6	7.0/39.1	7.0/70.3	3.3/29.8	7.0/44.2	53.7
14	3.7/11.9	8.2/28.4	6.8/30.3	8.8/42.3	3.4/198.3	239.3
15	11.4/101.4	2.4/37.1	3.9/51.0	9.3/90.5	6.5/41.1	36.6
16	8.3/36.3	3.0/13.0	2.9/68.0	3.0/42.1	7.1/13.9	35.9
HorseNumber	1 to 2,000	2,001 to 4,000	4,001 to 6,000	6,001 to 8,000	8,001 to 10,000	I_B
1	6.6/62.2	8./65.4	8.5/82.2	7.5/68.9	8.2/72.3	105.2
2	8.5/87.2	8.3/84.4	7.4/70.7	8.1/81.2	8.5/72.4	81.0
3	6.7/60.9	6.9/60.6	8.4/90.0	8.1/66.9	7.5/77.6	99.6
4	9.0/94.9	8.6/104.9	8.1/85.4	7.8/61.9	8.8/83.9	65.2
5	8.7/82.3	8.9/103.1	9.1/78.8	8.1/79.6	8.0/76.9	93.4
6	8.0/71.8	8.6/89.6	8.6/92.8	7.5/87.2	7.8/75.9	105.7
7	8.7/76.0	7.6/83.9	7.1/70.5	8.0/72.1	8.2/78.7	92.7
8	7.5/61.5	7.4/66.	8.8/83.1	8.6/89.8	6.7/53.2	86.5
9	7./76.4	6.9/75.6	5.9/51.7	6.9/66.4	6.9/56.5	67.7
10	7.8/84.5	7.9/84.6	7.0/71.5	7.6/66.5	6.6/78.4	78.7
11	8.1/78.2	6.6/59.9	6.4/88.6	7.6/68.5	8.3/90.9	76.6
12	7.6/89.7	7.3/91.7	6.3/70.	7.8/86.9	7.1/60.3	67.2
13	6.4/47.5	6.1/69.	6.6/89.6	6.1/92.5	6.8/101.5	145.0
14	5.7/49.7	5.2/57.4	6.8/80.3	5.7/65.	5.1/66.7	115.4
15	5.8/73.3	6.6/74.6	5.8/91.2	6.5/66.6	6.5/77.9	90.8
16	4.2/45.3	6.8/103.1	7.4/70.6	5.4/75.8	5.4/67.2	148.5

Table 9. Results of rank by number of horses

Rank	Total	6	7	8	9	10	11
1	34.4/81.5	47.7/76.7	47.5/86.4	38.4/73.9	41.9/81.2	41.5/86.3	35.9/77.7
2	19.4/79.7	25.6/94.2	24.3/87.9	25.6/92.4	20.8/74.5	19.9/74.0	20.7/83.7
3	12.4/81.7	9.3/62.1	11.9/65.6	12.8/83.0	14.2/90.1	12.1/77.5	12.7/83.4
4	8.7/80.9	10.5/119.7	5.9/46.9	8.0/70.3	9.1/101.2	8.5/72.1	9.4/84.8
5	6.3/79.2	4.7/76.2	6.9/80.2	6.9/91.7	4.2/46.2	4.9/63.6	5.8/76.9
6	5./82.4	2.3/74.9	3./50.3	5.2/97.3	3.8/63.0	4.9/99.6	5.1/88.0
7	3.7/75.3		0.5/11.7	1.7/41.9	4./95.	3./67.5	3.7/83.8
8	2.8/73.8			1.4/60.8	1.4/69.7	2.7/101.2	3.1/88.2
9	2.1/68.9				0.8/67.7	1.8/77.9	1.7/61.1
10	2.1/82.7					0.6/47.6	1.1/63.0
11	1.6/72.1						0.8/63.3
Races	10,000	86	202	289	506	668	830

Rank	Total	12	13	14	15	16
1	34.4/81.5	35./79.	31.8/74.0	33.3/81.2	32.2/82.1	31.7/85.2
2	19.4/79.7	22.2/89.5	19.6/80.7	16.4/69.1	20.8/87.0	17.2/75.7
3	12.4/81.7	12.3/81.9	14.0/90.5	13.4/90.7	11.0/72.8	12.0/80.0
4	8.7/80.9	9.3/87.2	7.2/70.5	11.0/99.1	8.2/76.9	8.3/77.4
5	6.3/79.2	5.4/69.3	7.6/96.0	5.8/75.0	7.0/82.7	6.8/84.7
6	5.0/82.4	5.5/91.8	5.7/98.4	5.6/85.3	4.8/70.3	4.9/77.5
7	3.7/75.3	2.6/48.6	3.4/67.0	4.3/80.0	3.7/76.2	4.4/87.6
8	2.8/73.8	2.2/71.6	2.4/66.8	2.7/60.6	2.9/65.2	3.3/77.6
9	2.1/68.9	1.4/45.1	2.0/59.7	1.7/61.5	2.2/62.3	3.0/85.6
10	2.1/82.7	1.8/90.9	1.7/68.2	1.9/71.2	2.5/94.7	2.8/96.8
11	1.6/72.1	1.4/76.8	1.8/105.8	1.4/56.0	1.9/80.5	1.7/63.5
12	1.3/73.4	1.0/74.2	1.6/110.7	0.9/54.9	1.4/66.7	1.4/70.2
13	1.0/88.4		1.4/193.0	1.2/106.5	1.1/71.6	0.8/54.1
14	0.7/61.0			0.5/49.4	0.2/26.0	1.0/81.3
15	0.3/33.6				0.2/33.2	0.4/33.7
16	0.4/80.3					0.4/80.3
Races	10,000	927	1023	1081	1381	3007

Table 10. Results of horse number by number of horses

HorseNumber	Total	6	7	8	9	10	11
1	7.7/70.2	18.6/89.5	12.4/38.7	11.1/52.3	8.7/54.9	10.9/80.8	9.2/74.8
2	8.2/79.2	14.0/46.5	19.3/80.1	10.7/56.4	11.5/79.2	9.3/70.8	10.6/98.2
3	7.5/71.2	16.3/117.1	16.3/76.1	9.3/51.1	10.9/86.1	8.7/57.6	8.0/60.5
4	8.5/86.2	15.1/73.8	12.9/55.8	14.9/75.1	12.3/81.4	10.5/86.0	11.0/100.2
5	8.5/84.2	12.8/29.9	17.3/75.0	15.2/143.3	12.3/67.8	11.7/108.7	8.9/64.9
6	8.1/83.4	23.3/146.9	9.9/35.9	10.7/55.3	10.7/72.6	9.4/93.0	8.7/98.7
7	7.9/76.3		11.9/67.3	13.1/68.5	11.9/115.1	10.2/65.9	8.8/68.6
8	7.8/70.7			14.9/109.4	10.3/59.6	11.1/80.7	8.9/79.5
9	6.7/65.3				11.7/71.7	8.8/59.5	7.1/66.7
10	7.4/77.1					9.4/64.0	8.9/70.5
11	7.4/77.2						10.0/71.3

HorseNumber	Total	12	13	14	15	16
1	7.7/70.2	7.7/67.5	6.0/77.4	7.8/87.0	6.0/52.4	6.9/72.9
2	8.2/79.2	7.3/59.8	7.5/87.8	7.2/91.9	8.3/85.3	6.2/74.5
3	7.5/71.2	7.9/51.7	7.1/84.2	6.6/47.9	6.7/62.1	6.3/89.0
4	8.5/86.2	8.2/120.1	9.9/123.9	6.7/49.7	7.1/80.1	6.4/79.3
5	8.5/84.2	7.8/53.1	9.3/103.8	8.0/96.1	6.7/69.8	6.7/88.5
6	8.1/83.4	8.0/71.3	10.4/124.4	7.4/79.2	6.6/66.7	6.5/82.
7	7.9/76.3	9.4/85.0	7.5/80.3	7.5/67.5	5.5/41.2	6.7/90.6
8	7.8/70.7	9.2/65.5	7.5/76.6	5.5/35.5	7.3/81.1	6.5/71.8
9	6.7/65.3	7.4/75.9	5.2/68.2	6.8/78.7	5.9/54.6	5.9/60.9
10	7.4/77.1	9.6/91.5	7.0/84.6	8.0/76.8	5.6/66.	6.6/80.2
11	7.4/77.2	9.2/66.7	9.0/121.8	6.3/69.9	6.4/70.4	6.5/72.5
12	7.2/79.7	8.4/97.9	7.9/82.7	10.7/121.3	7.2/84.7	5.3/55.7
13	6.4/80.0		5.7/65.7	6.3/79.7	7.6/80.7	6.1/84.8
14	5.7/63.8			5.3/59.2	5.9/62.8	5.8/65.9
15	6.2/76.7				7.2/90.6	5.8/70.3
16	5.9/72.3					5.9/72.3

Table 11. Losing/winning streak of rank

Rank	Losing Streak				Wining Streak			
	Min	Max	Average	stdev	Min	Max	Average	stdev
1	0	24	1.91	2.38	0	8	.52	.89
2	0	37	4.14	4.61	0	5	.24	.55
3	0	61	7.05	7.89	0	4	.14	.41
4	0	78	10.54	10.85	0	4	.09	.33
5	0	85	14.8	14.86	0	2	.07	.27
6	0	127	19.	19.84	0	2	.05	.23
7	0	171	26.16	26.15	0	3	.04	.2
8	0	192	34.97	34.63	0	2	.03	.17
9	0	248	45.42	46.42	0	4	.02	.15
10	0	345	46.95	52.51	0	2	.02	.15
11	0	291	61.98	58.05	0	2	.02	.13
12	0	363	75.49	73.01	0	2	.01	.12
13	0	354	94.49	91.98	0	2	.01	.1
14	2	606	142.95	138.16	0	1	.01	.08
15	5	852	291.6	243.44	0	1	.	.06
16	15	787	213.86	198.76	0	1	.	.07

Table 12. Losing/winning streak of horse number

Horse Number	Losing Streak				Winning Streak			
	Min	Max	Average	Stdev	Min	Max	Average	Stdev
1	0	83	11.94	13.04	0	3	.08	.3
2	0	69	11.26	11.67	0	3	.09	.31
3	0	86	12.3	12.41	0	3	.08	.3
4	0	80	10.82	10.59	0	3	.09	.31
5	0	81	10.71	10.73	0	4	.09	.32
6	0	77	11.38	11.8	0	4	.09	.31
7	0	81	11.6	12.12	0	3	.09	.3
8	0	85	11.78	12.27	0	3	.08	.3
9	0	106	13.86	15.05	0	3	.07	.28
10	0	94	12.53	12.48	0	3	.08	.29
11	0	78	12.48	12.95	0	3	.08	.29
12	0	107	12.84	13.32	0	3	.08	.29
13	0	88	14.68	15.37	0	3	.07	.27
14	0	114	16.53	17.71	0	3	.06	.26
15	0	90	14.96	15.15	0	3	.07	.26
16	0	79	15.99	15.12	0	2	.06	.26

In Sections 3 to 7, we will use data0. For ensuring the validation of the models, I constructed more racing data through 100,000 (10,000×10) races. These data are referred to as data1 to data10 and will be used in Section 8 and 10. The results are shown in Tables 13 and 14. Comparing with Tables 7 and 8, it can be seen that hit rates and recovery rates are relatively stable. That is, the differences between data0 and data1 through data10 are small, especially in the top ranks.

Table 13. Results of other 100,000 races of horse rank

Rank		R1	R2	R3	R4	R5	R6	R7	R8
data0		34.4/81.5	19.4/79.7	12.4/81.7	8.7/80.9	6.3/79.2	5.0/82.4	3.7/75.3	2.8/73.8
For Test	data1	34.9/81.9	18.5/77.5	12.8/83.0	8.5/79.6	6.3/79.9	4.9/78.5	3.7/79.2	2.8/70.3
	data2	33.5/78.9	19.7/81.9	12.8/82.6	8.8/83.0	6.5/81.8	4.9/80.7	3.7/77.6	3.2/77.7
	data3	33.4/79.3	20.0/83.8	12.3/79.7	8.3/78.0	7.0/86.6	4.7/77.1	3.3/72.0	3.0/79.7
	data4	35.4/83.3	19.5/80.1	12.0/78.2	8.1/76.6	6.3/82.8	4.8/75.2	3.7/77.7	2.9/75.9
	data5	34.0/79.9	19.5/82.4	11.8/76.3	8.9/83.8	6.8/84.4	4.8/79.3	4.0/84.1	2.8/70.1
	data6	34.9/82.0	19.3/81.5	12.7/83.7	8.2/75.7	6.3/78.6	4.7/77.0	3.9/80.5	2.5/63.2
	data7	34.6/81.8	19.9/82.8	12.2/79.0	8.2/78.0	5.9/72.5	4.9/80.3	4.1/85.2	2.8/73.4
	data8	34.2/80.9	18.6/76.6	12.8/84.1	8.3/76.5	6.5/81.4	5.0/79.1	3.9/78.1	3.0/78.1
	data9	34.5/81.5	19.4/81.0	12.2/79.6	8.7/81.0	6.0/74.4	4.7/76.1	3.7/77.3	3.1/82.3
	data10	33.2/78.3	20.0/84.0	12.6/82.3	9.0/84.2	6.5/81.6	4.5/74.0	4.0/84.6	3.3/88.2
I_Y		96.08	96.05	93.39	93.56	91.65	89.72	95.72	85.54
Rank		R9	R10	R11	R12	R13	R14	R15	R16
data0		2.1/68.9	2.1/82.7	1.6/72.1	1.3/73.4	1.0/88.4	0.7/61.0	0.3/33.6	0.4/80.3
For Test	data1	2.7/87.0	2.0/73.3	1.5/70.9	1.1/59.3	1.0/62.6	0.8/71.5	0.3/25.7	0.5/82.5
	data2	2.3/72.3	1.7/65.1	1.6/75.5	1.2/71.4	1.0/67.5	0.7/59.3	0.4/45.1	0.5/79.2
	data3	2.5/78.2	2.1/82.2	1.7/82.7	1.5/83.6	1.0/77.3	0.8/59.8	0.6/61.6	0.3/45.7
	data4	2.4/76.1	1.9/74.9	1.5/75.9	1.2/70.4	1.0/66.6	0.5/44.9	0.5/55.9	0.4/55.1
	data5	2.4/74.0	1.9/73.6	1.5/69.7	1.2/67.0	1.0/65.8	0.7/58.6	0.7/82.0	0.5/70.8
	data6	2.7/84.8	1.9/75.2	1.3/62.1	1.2/66.9	1.0/69.5	0.5/50.6	0.5/60.0	0.2/43.8
	data7	2.2/67.6	2.0/87.1	1.5/70.3	1.3/69.6	1.1/80.3	0.9/75.2	0.5/57.9	0.3/57.6
	data8	2.5/82.5	1.9/74.3	1.7/78.0	1.4/82.9	0.9/69.1	0.9/80.8	0.6/71.7	0.4/69.9
	data9	2.7/87.9	1.8/74.6	1.3/59.7	1.2/72.2	1.0/71.8	1.0/83.9	0.7/67.5	0.4/74.6
	data10	2.3/72.2	1.8/67.2	1.5/70.5	1.1/54.9	0.9/68.1	0.6/56.4	0.7/67.6	0.2/33.6
I_Y		98.05	78.70	82.78	74.73	70.84	73.60	76.49	41.90

I_Y: Ratio of the data0 to the minimum of data1 through data10.

Table 14. Results of other 100,000 races of horse number

Horse Number		N1	N2	N3	N4	N5	N6	N7	N8
data0		7.72/70.2	8.2/79.2	7.5/71.2	8.5/86.2	8.5/84.2	8.1/83.4	7.9/76.3	7.8/70.7
For Test	data1	7.7/82.8	7.8/72.3	7.8/69.1	8.8/88.5	8.0/74.3	7.9/85.9	8.1/77.7	7.9/71.6
	data2	7.3/62.9	7.6/67.6	8.0/76.4	8.1/72.4	8.0/87.0	8.7/85.1	8.6/89.6	8.1/79.7
	data3	7.7/71.7	8.3/74.4	7.7/80.5	8.4/80.0	8.2/86.3	8.2/82.3	8.2/85.5	7.2/75.4
	data4	8.0/82.5	8.0/81.4	7.7/71.6	9.0/85.0	7.8/67.1	8.2/74.6	8.1/75.3	7.8/75.3
	data5	7.3/65.7	8.1/76.9	7.9/77.3	8.7/86.0	8.3/81.5	7.9/74.2	7.7/74.7	8.2/81.0
	data6	7.3/66.1	8.0/73.4	7.7/70.1	9.3/89.7	8.0/69.7	8.3/83.0	8.3/85.0	8.2/71.0
	data7	7.2/66.6	8.1/82.4	7.4/70.3	8.8/89.5	8.2/76.3	8.1/85.2	8.3/85.6	8.2/78.5
	data8	7.4/80.1	8.8/91.1	8.1/82.0	8.4/86.8	7.9/73.1	7.5/75.3	8.1/78.5	8.1/86.5
	data9	7.4/72.6	8.6/79.9	7.7/74.3	8.4/76.5	8.6/91.8	8.2/80.4	8.0/76.2	8.2/74.3
	data10	7.5/73.6	8.4/82.1	7.8/71.2	8.9/81.8	8.4/79.2	8.0/70.5	8.5/78.3	7.3/77.6
I_Y		89.63	85.37	97.12	83.99	79.71	84.52	97.92	100.43
Horse Number		N9	N10	N11	N12	N13	N14	N15	N16
data0		6.7/65.3	7.4/77.1	7.4/77.2	7.2/79.7	6.4/80.0	5.7/63.8	6.2/76.7	5.9/72.3
For Test	data1	7.1/70.8	7.8/76.4	7.1/84.8	6.4/61.4	6.4/65.6	5.9/62.2	5.5/54.9	6.3/73.7
	data2	6.7/65.1	7.9/78.9	6.7/76.4	7.4/78.1	6.1/63.8	6.1/65.4	6.1/75.7	5.7/72.3
	data3	6.7/69.3	7.9/80.4	7.0/72.9	7.3/93.1	6.1/66.9	6.5/69.1	5.7/68.7	6.1/76.1
	data4	7.1/61.6	7.4/81.2	6.9/69.4	6.73/76.7	6.4/70.0	6.1/64.9	5.5/68.4	6.2/77.4
	data5	6.9/77.1	8.1/85.4	6.3/59.6	7.6/78.7	6.3/62.3	5.8/74.7	6.3/83.9	6.3/77.7
	data6	6.5/57.8	7.3/77.6	7.2/72.8	7.6/79.1	5.3/61.6	6.1/70.5	6.0/70.8	6.2/74.6
	data7	6.8/67.4	7.8/76.3	7.1/73.0	7.0/73.4	5.9/76.1	6.0/69.1	6.3/64.3	5.4/81.6
	data8	7.0/73.6	8.0/85.6	6.4/72.7	6.8/66.3	6.5/70.3	6.1/68.9	6.1/76.2	6.4/71.7
	data9	7.3/80.6	7.4/77.1	6.8/80.9	7.2/84.8	6.0/69.4	5.8/64.9	5.8/65.3	4.8/67.5
	data10	6.6/67.4	7.4/77.6	6.5/67.3	7.2/71.9	6.6/68.7	6.0/76.5	5.5/67.8	6.2/85.6
I_Y		88.55	98.91	77.27	77.10	76.97	97.50	71.59	93.37

Section 3 Definitions and the standard model

3.1 Definitions

We define the following constants and variables:

Definition of Constants

We define the following predetermined constants:

a_{init}: initial input amount. In Japan, minimum betting amount is 100 yen, hence, in this paper, I set 100 yen simply. When obtaining refunds which is larger than uncollected amount, input amount is reset to this value.

b: increase coefficient. Input amount is computed as round up of ratio of uncollected amount and this value.

n: number of races.

ν: Number of race per 1 period. We can freely define 1 period to 1 day, 2 days, 1 week, 1 month etc. We restart betting the first race of the next period, that is, restart betting when mod(n,ν)=1 where mod(y, x) means remainder when y divided by x. (Section 5.3)

ξ: Starting odds of the most popular horse. We start betting the most popular horse when odds are this value or under. (Section 5.4)

q: Number of candidates of b. (Section 6)

${}^{i}b$: i-th candidate of b,

b^: Standard value of b, ${}^{i}b$ is set as rate of this value. In this paper, b^ is set to (median-100).

${}_{j}e$: Coefficient which moves to next larger ${}^{j}b$ (j=1, ... , q-1).

R^{start}: Rank while uncollected amount is small. (Section 7)

R^{max}: Maximum rank when uncollected amount is large.

${}^{j}b_r$: Candidate of b for rank r. (j=1, ... , q-1)

${}_{j}f_r$: Coefficient which moves to next larger ${}^{j}b_r$ (j=1, ..., q-1)

${}_{i}b$: b for i-th rank horse.

AG: Parameter that controls when to abandon increasing the input amount and when to reset the uncollected amount to 0. (Section 9)

d^{sta}: Starting value of the divided uncollected amount.

d^{back}: Threshold set to the minimum value for the uncollected amount while d_n (divided uncollected amount) remains.

c_ln: Coefficient for computing line number. (Section 10.1.4)

max_div: Maximum division number (line number) of uncollected amount.

$b^{\#}$: Standard value of b for division, we divide uncollected amount.

b_idx: Index for computing b.

Definition of Variables

Variables are defined as follows. These value changes as races progress.

$g^r{}_n$: horse number of r-th rank horse of n-th race.

o_n: odds of n-th race at the betting time. Refunds may change after betting.

$o^r{}_n$: odds of r-th rank horse of n-th race at the betting time. Refunds may change after betting.

N_n: Number of horses of n-th race.

W_n: Number of winning horses of n-th race.

G_n: Refund of n-th race for 100 yen.

N_n: Number of horses of n-th race.

a_n: input amount of n-th race. Input amount computed from after-race uncollected amount and input increase decision value. a_1 is set to a_{init} .

t_n: before-race uncollected amount of n-th race. The sum of after-race uncollected amount of (n-1)-th race and input amount.

r_n: refunds of n-th race for 100 yen betting. When losing, this value is zero, when winning, r_n=G_n.

s_n: refunds of n-th race for input amount a_n . When a_n=100, s_n=r_n .

u_n : after-race uncollected amount. When losing, this value is before-race uncollected amount. When obtaining smaller than before-race uncollected value, the differences are remained. When obtaining larger than before-race uncollected value, reset to 0.

race_no: Race number in the period. (Section 5.3)

σ_{vn}: Total input amount of the day. (We use the condition σ_{vn}=0 at Section 5.3)

h_{rn}: Rank of betting horse of n-th race (Section 7)

d_n : Divided uncollected amount. (Section 9).

i : Rank of betting horse or line number. (Section 10).

next_i: Line index of the next race.

u_{ni} : after-race uncollected amount for line i.

a_{ni} : Input amount for line i.

line_num : Number of betting horses include stopped betting horses, or number of lines when betting one horse.

Definition of Ancillary observation value

We use following observation values.

$A_{\max}$: Maximum input amount.

c_b: Betting count.

r_u: Update rate, that is, (count of profit update) ÷(betting count).

r_h: Hitting rate.

r_r: Recovery rate.

P_n: Total profit. Difference of investment and acquisition.

K_n: Recovery rate.

$P_{n\_\max}$: Maximum profit from P_1 to P_n.

$P_{n\_\min}$: Minimum profit from P_1 to P_n.

K_n: Recovery rate. It is ratio of Q_n and n .

$R_{(p\%)}$: p-percentile of refunds for 100 yen betting.

$R^0{}_{(p\%)}$: p-percentile of refunds computed losing ticket as 0 for 100 yen betting.

$O_{(p\%)}$: p-percentile of odds for 100 yen betting.

I_z: Ratio of the maximum $A_{\max}$ of data0 to the maximum of $A_{\max}$ of data1 through data10. (Section 8)

$A^{\wedge}{}_{\max}$: Maximum input amount at the one race. (Section 10)

$P^{\wedge}{}_n$: Sum of total profit.

$P^{\wedge☆}{}_n$: Best value of sum of total profit.

$P^{\mathrm{tax}}{}_n$: Total profit for computing tax. (Section 11)

Notation of betting Strategy

Rr: Betting r-th rank(popular) horse.

Nn: Betting horse number n.

3.2 Steps of the standard model

For this model, we perform the following steps iteratively:

STEPS of the standard model:

STEP0 Set a_{init}=100, n=1, a_n=a_{init}, u_0=0.

STEP1 Buy horse ticket a_n

STEP2 Set $t_n=u_{(n\text{-}1)}+a_n$

STEP3 After the n-th race, confirm returns r_n

Compute s_n as returns (if loss k s_n=0 else $s_n=r_n\times a_n\div 100$)

STEP4 Compute u_n. If $t_n<s_n$ then u_n=0. If $t_n>s_n$ then $u_n=t_n-s_n$.

STEP5 Set $n=n+1$

STEP6 Compute a_n as follows:

If $u_{(n\text{-}1)}=0$ then $a_n=a_{init}$

(8) else $a_n=\lceil u_{(n\text{-}1)}\div b\rceil\times a_{init}$,

where [x] represents the round-up value of x.

STEP7 Go to STEP1

If b is 100, this model is the same as the standard doubling method (except that the first two bets are 100 yen each). However, in this paper, we can set b freely with consideration of the stochastic information inherent in the betting strategy's hit rates, recovery rates, and so on.

Table 15 shows examples. Cases (a) and (b) show the effect of b as b's value changes. For case (a), when b=100, the input amount, a_n, in the second race becomes double that of the previous race during a losing streak. The value of a_n for case (a) becomes larger in the 9th race than that for case (b). Cases (b) and (c) show examples of the significance of the timing of wins. In case (b), we obtain larger returns than with case (c), because a_n is increased by a losing streak. In this way, even if we set b to the same value, the returns may change according to when wins occur.

Table 15. An example of the standard model

case	b	n	0	1	2	3	4	5	6	7	8	9	10
(a)	100	a_n		100	100	200	400	800	1600	3200	6400	12800	100
		t_n		100	200	400	800	1600	3200	6400	12800	25600	100
		r_n		0	0	0	0	0	0	0	0	480	320
		s_n		0	0	0	0	0	0	0	0	61440	320
		u_n	0	100	200	400	800	1600	3200	6400	12800	0	0
		P_n		-100	-200	-400	-800	-1600	-3200	-6400	-12800	35840	36060
		K_n		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	240.0	240.3
(b)	350	a_n		100	100	100	100	200	200	300	400	500	100
		t_n		100	200	300	400	600	800	1100	1500	2000	100
		r_n		0	0	0	0	0	0	0	0	480	320
		s_n		0	0	0	0	0	0	0	0	2400	320
		u_n	0	100	200	300	400	600	800	1100	1500	0	0
		P_n		-100	-200	-300	-400	-600	-800	-1100	-1500	400	620
		K_n		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	120.0	129.5
(c)	350	a_n		100	100	100	100	100	100	100	100	200	200
		t_n		100	200	300	400	100	200	300	400	600	800
		r_n		0	0	0	480	0	0	0	0	0	320
		s_n		0	0	0	480	0	0	0	0	0	640
		u_n	0	100	200	300	0	100	200	300	400	600	160
		P_n		-100	-200	-300	80	-20	-120	-220	-320	-520	-80
		K_n		0.0	0.0	0.0	120.0	96.0	80.0	68.6	60.0	48.0	93.3

Table 15 shows how the increase coefficient, *b*, affects the recovery rate and the funds (the maximum input amount and the uncollected amount). A small value for *b* results in a large recovery rate while creating the need for a large amount of funds. As a result, this may cause a lack of funds. How much is appropriate? The hint, I think, is to consider hit rate, percentage points, and the length of losing streaks. That is, when ρ is the hit rate, the probability that all the next *h* races will lose can be computed as $(1—\rho)^h$. When ρ is the probability that the returns are lower than $R^0{}_{(p\%)}$ (*p*-percentage points of returns computed losing tickets as 0), the probability that all the next *h* races' returns are lower than $R^0{}_{(p\%)}$ can be computed as $(1—\rho)^h$. Furthermore, from (8), there are cases where the uncollected amount cannot be zero, if returns exceed $b+100$ (100 is the minimum betting amount in Japan). So I propose to set *b* as follows for a predetermined *p*:

(9) $b{:=}R^0{}_{(p\%)} - 100$

or

(10) $b := R_{(p\%)} - 100$

Section 4 The one-horse one-line model

4.1 The standard model (Type I)

In this section, I propose two betting strategies. One strategy is based on betting on a fixed rank, and the other is based on betting on a fixed horse number.

4.1.1 Strategy for betting on a fixed rank (R*r* strategy)

With this strategy, we bet on a predetermined fixed rank horse. In Japan, we can obtain information for horses whose recovery rates exceed 80% by race type, racecourse, and race conditions. In addition, we can check the ranks and the odds before betting. As a method that does not require predictions, this is easy to apply.

Table 16 shows the first 16 betting races for the most popular horse (R1).

Table 16. First 16 betting races for R1

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$g^1{}_n\_o^1{}_n$	11_3.2	11_3.6	14_2.4	7_2.6	5_3.9	6_4.3	4_2.3	5_1.9	15_2.3	8_1.6	16_2.2	13_3.2	11_5.3	9_4.7	3_1.2	14_2.7
a_n	100	100	200	100	100	200	400	700	200	400	300	500	900	1,700	100	100
t_n	100	200	400	100	200	400	800	1,500	440	840	580	1,080	1,980	3,680	100	100
$W_n\_G_n$	13_1,040	14_390	14_250	5_350	11_450	16_450	2_830	5_180	10_760	8_140	15_970	2_830	5_2,840	9_450	3_120	9_2,220
s_n			500					1,260		560				7,650	120	
u_n	100	200		100	200	400	800	240	440	280	580	1,080	1,980			100
P_n	-100	-200	100		-100	-300	-700	-140	-340	-180	-480	-980	-1,880	4,070	4,090	3,990

For figures in Sections 4 to 7, the line graph and the left axis show the profits change, and the vertical bar graph and the right axis show the return for a 100 yen bet. We can discover how much of a return can be obtained, and how often returns occur. For figures in Sections 4 and 5, b is set to (median of refunds)–100 (see Table 17). For example, the median for the most popular horse is 220, so b is set to 120.

Fig. 1 shows examples of the standard one-horse one-line model by rank from the 1st to the 6th rank. In conditions where the hit rates are both high and low, we can see an upward trend over the long term. However, there are some deep icicle-shaped valleys. We would need to make large wagers (the input amount) exceeding 1 million yen at the points where these valleys appear. This is unrealistic, as we can't afford to bet such large amounts of money. Furthermore, if we did make such large wagers, the odds would be driven down, and we would not be able to acquire the expected return.

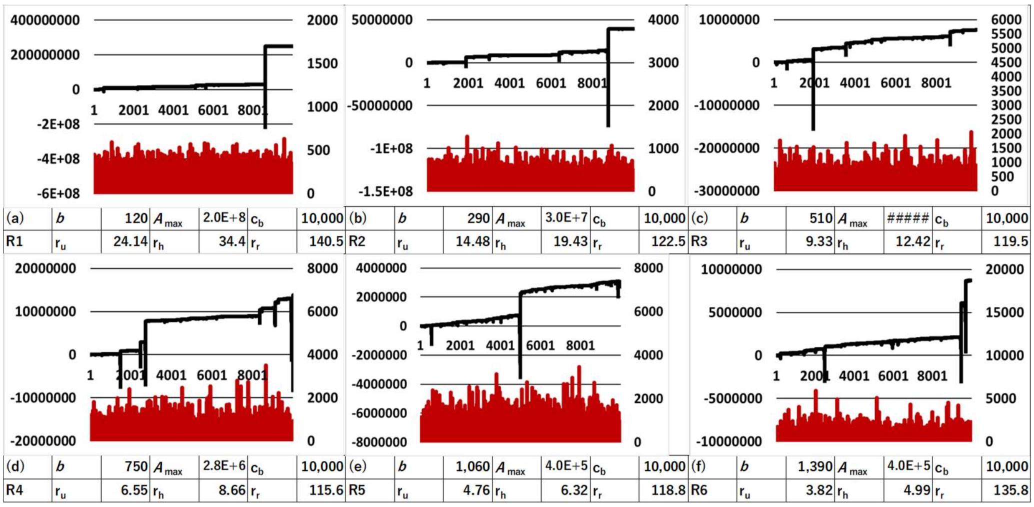


Fig. 1. Examples of profit change by rank.

Table 17. Parameters of Fig. 1, 3, 5, 6, 7, 8 and 9 for R*r* strategy

Rank	$R_{(50\%)}$	$R^0_{(\alpha\%)}$	α	b	Rank	$R_{(50\%)}$	$R^0_{(\alpha\%)}$	α	b
1	220	220	82	120	4	850	850	95.7	750
2	390	390	90.5	290	5	1160	1160	96.85	1,060
3	610	610	93.7	510	6	1490	1490	97.5	1,390

4.1.2 Strategy for betting on a fixed horse number (N*n* strategy)

With this strategy, we bet on a predetermined fixed horse number. In Japan, we can research which horse number has the advantage by race conditions such as the racecourse, distance, surface (turf/dirt), and so on.

Fig. 2 shows examples by horse number. Table 18 shows where b (b=9,900) is positioned in the distribution of refunds. We can see that the interval of returns exceeding 9,999 occurs irregularly. Under the assumption that the odds are not changed by our wagers, when u_n is large, the return s_n becomes very large. In Section 5, we set b=9,900.

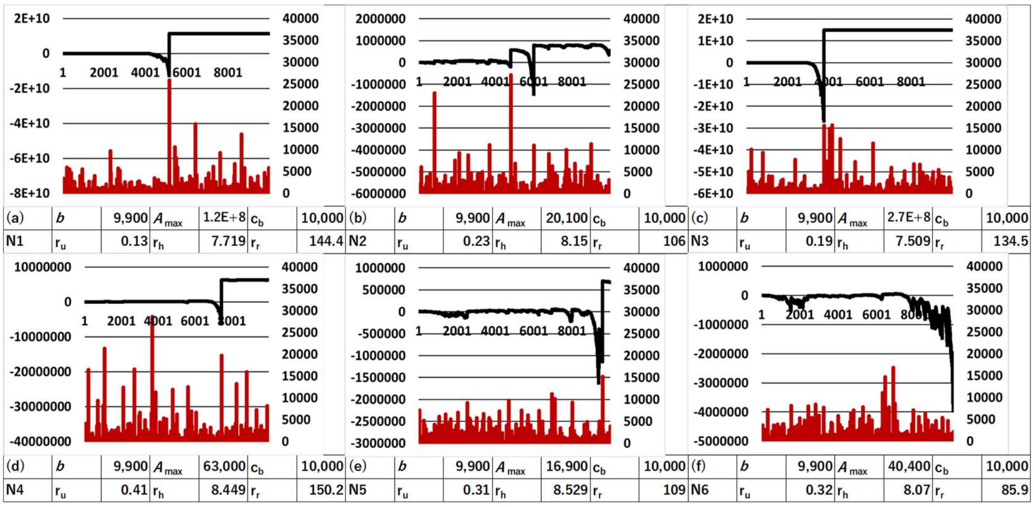


Fig. 2. Examples of profit change by horse number.

Table 18: Parameters of Fig. 2, 3, 5, 6, 7, 8 and 9 for Nn strategy

Number	b	$R_{(\alpha\%)}$	α	$R^0_{(\beta\%)}$	β	Number	b	$R_{(\alpha\%)}$	α	$R^0_{(\beta\%)}$	β
1	9,900	10,007.38	99.53	10,002.22	99.9637	4	9,900	10,001.57	98.734	10,003.69	99.8932
2	9,900	10,008.72	99.37	10,007.71	99.9487	5	9,900	10,004.37	99.705	10,001.71	99.9748
3	9,900	10,006.23	99.173	10,008.44	99.938	6	9,900	10,004.99	99.6973	10,005.01	99.9756

4.2 Results over a short time period

Fig. 3 shows the good and bad time periods of Fig. 1(a) and 2(d).

3(a) shows the first 250 races of Fig. 1(a). Over a short period, we can see a smoother upward trend, but at the deepest valley (the 33rd race), we needed to bet 27,000 yen.

3(b) and 3(c) show the worst and the second worst cases, respectively. 3(b) shows the change of profits from the 8,583rd race to the 8,621st race of Fig. 1(a). At the 8,587th race, u_n is 0, and following that, there were no wins with returns over 220 (=b+100) yen until the 8,614th race. At race 8,589, r_n=170 (this win is not enough to clear u_n), and all other races were lost. Then, at the 8,614th race, the betting amount reaches 200,675,400 yen and r_n=330. This results in obtaining a return of s_n=662,228,820.

Similarly, in 3(c), at the 5,580th race, u_n is zero. At the 5,588th race, r_n=190 (s_n=8,360), and u_n= 1,240 remains. At race 5,589, r_n=160 (s_n=1,760), and u_n=580 remains. At race 5,601, r_n=140 (s_n=553,840), and u_n=316,440 remains. At race 5,606, r_n=190 (s_n=5,660,670), and u_n=893,770 remains. At race 5,609, r_n=190 (s_n=4,756,840), and u_n=751,030 remains. All other races were lost. Finally, at the 5,615th race, the input amount reaches 12,963,200 yen and r_n=240, and we obtain a return of s_n=31,111,680. These examples show that not only losing streaks, but also streaks of returns that are less than b+100, are terrible in this model.

Similar to the R*r* strategy, the N*n* strategy also experiences both good and bad time periods, as shown in Fig. 3 (d)-(f). From (e) and (f), we can see that it is difficult to recover from those times when we cannot acquire an r_n larger than b+100(=10,000).

These results show that even if the results at a certain point are good, a model that is created based on limited historical data will eventually become a nightmare. When I started this research, I calculated a value for b that seemed reasonable based on the results of several hundred races. After putting it into practice, I ended up losing more than 100,000 yen.

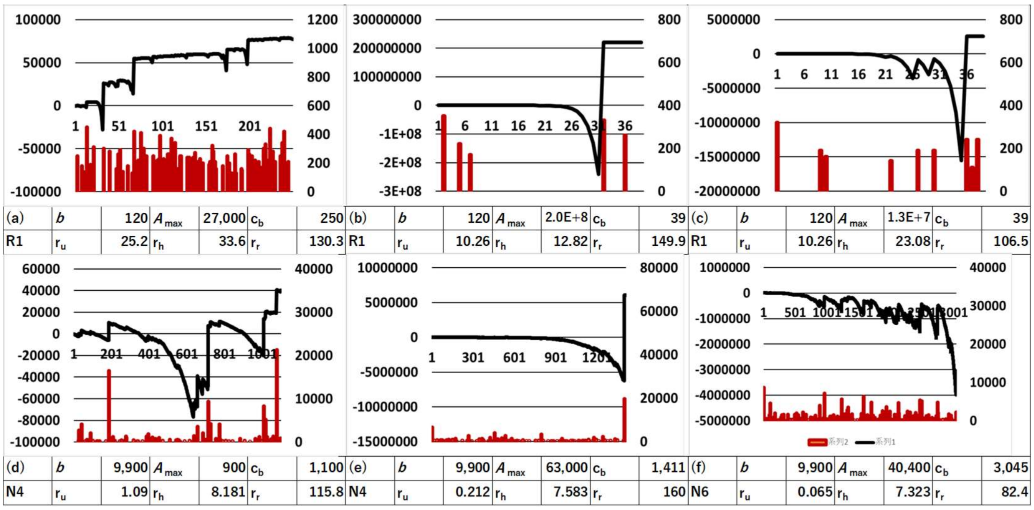


Fig. 3. Good and bad periods of Fig. 1(a) and Fig. 2(d).

4.3. Difficulty of setting the increase coefficient

Table 19 and Fig. 4 show the results of using different values of b with the same racing data as those in Fig. 1(a), Fig. 1(f) and Fig. 2(d). This shows that we can obtain a large recovery rate when b is small, but if b is small, bankruptcy is threatened. On the other hand, when b is large, recovery rates are small, sometimes even less than 100. This makes doubling meaningless, and sufficient consideration cannot be made with only this information. For now, it’s better to give up trying to find an appropriate value for b and instead consider a better method.

Table 19. Examples of the effect of varying *b*

R1				R6				N4			
b	A_{max}	r_u	r_r	*b*	A_{max}	r_u	r_r	*b*	A_{max}	r_u	r_r
80	2.2E+10	29.2	179.3	390	2.5E+14	4.99	401.9	2,400	5.9E+7	1.2	427.7
120	2.0E+8	24.14	140.5	640	6.1E+9	4.98	255.3	3,900	1.3E+7	0.67	337.4
160	8.1E+6	20.4	109.2	890	4.5E+7	4.76	190.3	5,400	4.9E+6	0.52	258.8
200	7.6E+6	15.61	101.8	1,140	2.5E+6	4.4	161.6	6,900	5.9E+5	0.51	206.5
240	6.6E+6	12.73	100.7	1,390	4.0E+5	3.82	135.8	8,400	2.0E+5	0.399	174.3
280	4.5E+6	9.94	100.4	1,640	1.1E+5	3.33	118	9,900	63,000	0.41	150.2
320	2.5E+6	7.3	100.2	1,890	58,300	2.86	109.3	11,400	28,600	0.28	133.4
360	5.6E+6	5.66	100.1	2,140	1.9E+5	2.44	102.1	12,900	16,800	0.27	122.1
400	4.0E+7	3.969	100	2,390	6.3E+5	2.2	100.3	14,400	11,100	0.23	115
440	1.6E+8	2.49	100	2,640	6.1E+5	1.85	100.1	15,900	11,000	0.18	110.1
480	1.6E+10	1.15	100	2,890	3.6E+6	1.65	100.5	17,400	6,900	0.199	105.4
520	5.2E+13	0.12	100	3,140	5.0E+6	1.4	101.1	18,900	6,600	0.19	101.9
560	3.0E+20	0	99.9	3,390	6.9E+6	1.21	101.4	20,400	5,000	0.13	99.6

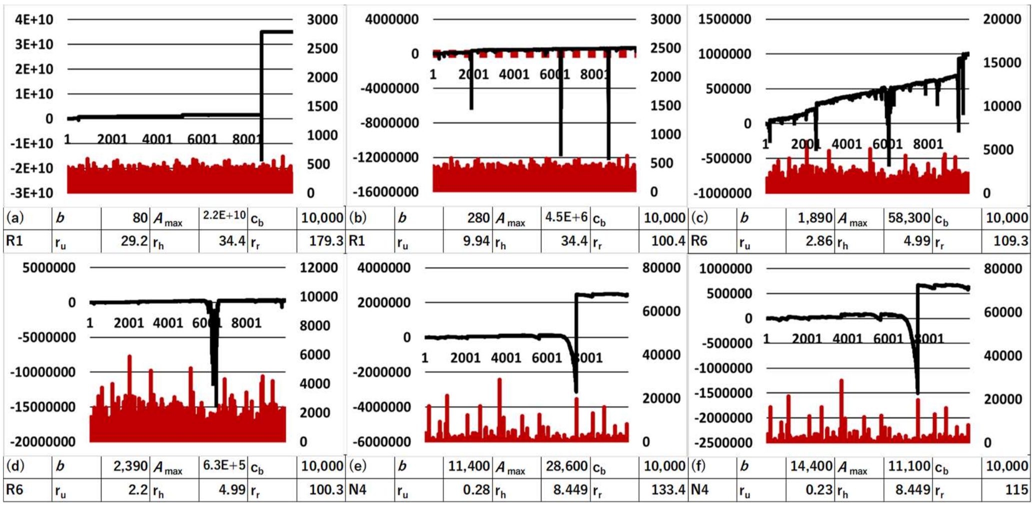


Fig. 4. Change of trend by some *b* of Fig. 1(a), 1(f) and 2(d).

Section 5 Reduction of the betting count

One effective method to reduce the risk that the input amount (the amount wagered) becomes too large is to reduce the betting count. This term refers to the number of bets. In this section, several methods for reducing the betting count are introduced.

5.1 Skip low-odds races when the uncollected amount is large (Type II)

As shown in Fig. 3(c) and 3(f), when u_n is large, u_n cannot be cleared even if the wagered horse wins, because the values of r_n and s_n are small. So, an effective strategy is to only bet on races whose odds are higher than b+100. We apply this strategy when the value of u_n is large. We can replace (8) by the following computation:

$$\text{If } (u_{(n\text{-}1)}<b \text{ and } o_n<(b+100)\div 100\ ) \text{ or } o_n \geqq (b+100)\div 100$$

(11) $$\text{then } (a_n=\lceil u_{(n\text{-}1)}\div b\rceil\times a_{init} \text{ If } a_n=0 \text{ then } a_n=a_{init}\,)$$

$$\text{Else } a_n=0$$

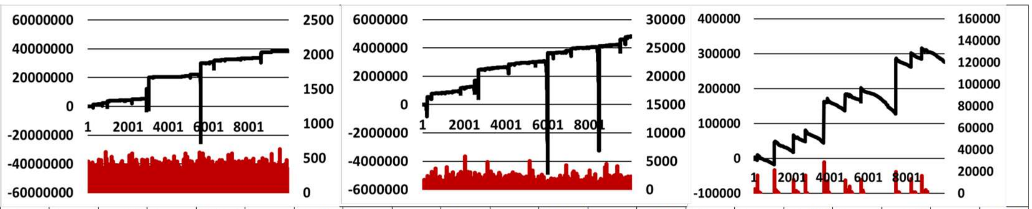
Table 20 shows the first 16 betting races. Compared with Table 16, we see that the 8th, 10th and 18th races are skipped.

Table 20. First 16 betting races of Type II

n	1	2	3	4	5	6	7	9	11	12	13	14	15	16	17	19
$g^1{}_n\_o^1{}_n$	11_3.2	11_3.6	14_2.4	7_2.6	5_3.9	6_4.3	4_2.3	15_2.3	16_2.2	13_3.2	11_5.3	9_4.7	3_1.2	14_2.7	1_2.4	4_4.5
a_n	100	100	200	100	100	200	400	700	1,300	2,400	4,400	8,000	100	100	100	200
t_n	100	200	400	100	200	400	800	1,500	2,800	5,200	9,600	17,600	100	100	200	400
$W_n\_G_n$	13_1,040	14_390	14_250	5_350	11_450	16_450	2_830	10_760	15_970	2_830	5_2,840	9_450	3_120	9_2,220	2_450	15_620
s_n			500									36,000	120			
u_n	100	200		100	200	400	800	1,500	2,800	5,200	9,600			100	200	400
P_n	-100	-200	100		-100	-300	-700	-1,400	-2,700	-5,100	-9,500	18,500	18,520	18,420	18,320	18,120

Fig. 5 shows some examples. Compared with Fig. 1(a), 1(f) and 2(d), the values of $A_{\max}$ for R1 and N4 are smaller, while the value of $A_{\max}$ for R6 is larger.

However, the total number of races where bets were placed is reduced by 20% or more. Furthermore, the difference between r_u and r_h becomes smaller for the R*r* strategy. This means we can increase the probability of clearing the uncollected amount (the accumulated losses) when the wagered horse wins.



(a)	*b*	120	A_{max}	3.9E+7	c_b	8,202	(b)	*b*	1,390	A_{max}	5.6E+5	c_b	7,758	(c)	*b*	9,900	A_{max}	800	c_b	3,376
R1	r_u	28.15	r_h	31.91	r_r	121.7	R6	r_u	4.253	r_h	4.421	r_r	119.9	N4	r_u	0.71	r_h	7.109	r_r	156.9

Fig. 5. Profit change of Type II.

5.2 Skip all low-odds races (Type III)

To reduce the betting count, let's consider the case where we only bet on races with high odds. Let's replace (8) as follows:

$$\text{If } o_n \geqq (b+100) \div 100$$

$$\text{then } (a_n = [u_{(n-1)} \div b] \times a_{init} \text{ If } a_n = 0 \text{ then } a_n = a_{init}) \quad (12)$$

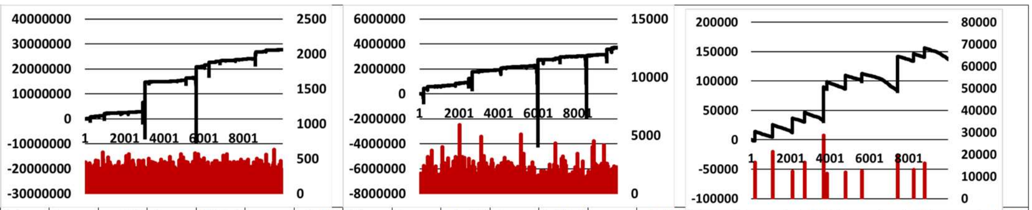
$$\text{Else } a_n = 0$$

Table 21 shows the first 16 betting races. Compared with Table 20, we skip only the 15th race in this example.

Table 21. First 16 betting races of Type III

n	1	2	3	4	5	6	7	9	11	12	13	14	16	17	19	20
$g^1{}_n\_o^1{}_n$	11_3.2	11_3.6	14_2.4	7_2.6	5_3.9	6_4.3	4_2.3	15_2.3	16_2.2	13_3.2	11_5.3	9_4.7	14_2.7	1_2.4	4_4.5	1_3.9
a_n	100	100	200	100	100	200	400	700	1,300	2,400	4,400	8,000	100	100	200	400
t_n	100	200	400	100	200	400	800	1,500	2,800	5,200	9,600	17,600	100	200	400	800
$W_n\_G_n$	13_1,040	14_390	14_250	5_350	11_450	16_450	2_830	10_760	15_970	2_830	5_2,840	9_450	9_2,220	2_450	15_620	3_880
s_n			500									36,000				
u_n	100	200		100	200	400	800	1,500	2,800	5,200	9,600		100	200	400	800
P_n	-100	-200	100		-100	-300	-700	-1,400	-2,700	-5,100	-9,500	18,500	18,400	18,300	18,100	17,700

Fig. 6 shows some examples. We were able to reduce the betting count. Furthermore, the value of $A_{\max}$ became smaller in all cases. While this may be an intuitive interpretation, this shows that it is better to bet when the odds are high, since there are more misses than hits.



(a)	b	120	$A_{\max}$	2.1E+7	c_b	6,920	(b)	b	1,390	$A_{\max}$	4.6E+5	c_b	6,476	(c)	b	9,900	$A_{\max}$	300	c_b	1,138
R1	r_u	27.99	r_h	29.31	r_r	123.5	R6	r_u	3.798	r_h	3.86	r_r	120.5	N4	r_u	0.966	r_h	0.966	r_r	201.3

Fig. 6. Profit change of Type III.

5.3 Skip all races after a win (Type IV)

Considering practical application, we want to avoid having to end the day with a negative profit. Following that, intuitively it seems effective to use a strategy of skipping all races after a win and resume betting from the next day's high-odds race. Let's consider replacing (8) as follows:

$$\begin{aligned}&\text{If } (o_n \geqq (b+100) \div 100 \text{ and } u_{(n-1)} > 0)\\ &\quad \text{or } (o_n \geqq (b+100) \div 100 \text{ and } u_{(n-1)} \leqq 0 \text{ and } \sigma_{\nu n}=0)\\ &\qquad \text{then } (a_n = [u_{(n-1)} \div b] \times a_{init} \text{ If } a_n=0 \text{ then } a_n = a_{init})\\ &\text{Else } a_n=0\end{aligned} \tag{13}$$

where $\sigma_{\nu n}$ is the total input amount of the predetermined period, such as one day, two days, one week, one month, one season, and so on. We can compute $\sigma_{\nu n}$ as follows:

(14)

Set race_no=if mod(n, ν) =0 then ν—1 else mod(n, ν)—1 end

$\sigma_{\nu n}=0$

For i=race_no to 1 step —1

$\sigma_{vn} = \sigma_{vn} + a_i$

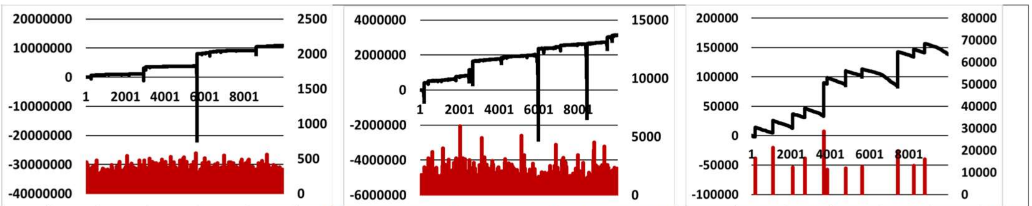
Next i

Table 22 shows the first 16 betting races in the case where ν=10. By this operation, we were able to skip many races.

Table 22. First 16 betting races of Type IV (ν=10)

n	1	2	3	11	12	13	14	22	31	32	33	41	44	45	46	47
$g^1_n\_o^1_n$	11_3.2	11_3.6	14_2.4	16_2.2	13_3.2	11_5.3	9_4.7	12_3.1	4_3.1	2_3.5	5_3	15_2.6	16_2.4	10_3.6	15_2.7	1_3.4
a_n	100	100	200	100	100	200	400	100	100	100	200	100	100	200	400	700
t_n	100	200	400	100	200	400	800	100	100	200	400	100	200	400	800	1,500
$W_n\_G_n$	13_1,040	14_390	14_250	15_970	2_830	5_2,840	9_450	12_310	5_7,510	6_360	5_300	4_4,110	15_2,060	5_430	6_3,650	6_1,120
s_n			500				1,800	310			600					
u_n	100	200		100	200	400			100	200		100	200	400	800	1,500
P_n	-100	-200	100		-100	-300	1,100	1,310	1,210	1,110	1,510	1,410	1,310	1,110	710	10

Fig. 7 shows some examples. We assume there are ν=10 races in a day. We start betting from the race whose odds are (b+100)÷100 or higher. If the wagered horse wins and u_n becomes 0, we skip all the following races for that day. If the wagered horse loses, we continue doubling with high-odds races. By this operation, we can reduce the betting count, and we also expect to reduce $A_{\max}$.



(a)	b	120	$A_{\max}$	2.1E+7	c_b	3,154	(b)	b	1,390	$A_{\max}$	3.5E+5	c_b	5,833	(c)	b	9,900	$A_{\max}$	300	c_b	1,131
R1	r_u	27.3	r_h	28.63	r_r	116.5	R6	r_u	3.823	r_h	3.891	r_r	121.2	N4	r_u	0.972	r_h	0.972	r_r	202.6

Fig. 7. Profit change of Type IV (ν=10).

5.4 Start with the most popular low-odds race (Type V)

As shown in Table 6, for the most popular horse, the hit rates of low-odds bets are higher. One effective strategy is to start betting on horses with low odds first. If the wagered horse wins, the recovery rate is rather small. However, we can avoid the risks of doubling. We can replace (8) by the following computation:

If ($o_n \geqq (b+100) \div 100$ and $u_{(n-1)} > 0$)

or ($o_n \leqq \xi$ and $u_{(n-1)} \leqq 0$ and $\sigma_n = 0$)

(15) then ($a_n = \lceil u_{(n-1)} \div b \rceil \times a_{init}$ If $a_n = 0$ then $a_n = a_{init}$)

Else $a_n = 0$

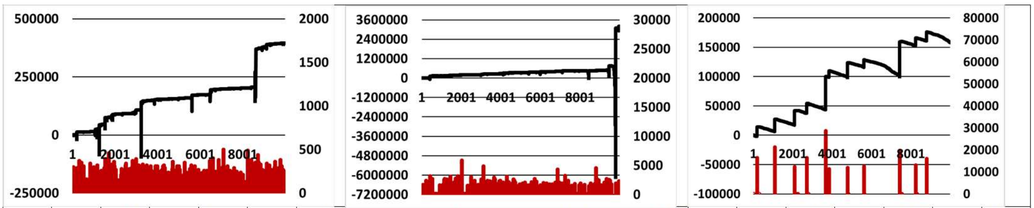
If $\sigma_{\nu n} = 0$, bet on the most popular horse.

Table 23 shows the first 16 betting races for the case ξ=1.5, ν=10. A race with odds of 1.5 or less may not occur for a while. In the meantime, we don't place bets, thereby reducing the betting count.

Table 23. First 16 betting races of Type V

n	15	28	31	32	33	43	44	45	46	47	49	50	55	66	80	104
$g^1{}_n\_o^1{}_n$	3_1.2	11_1.5	4_3.1	2_3.5	5_3	7_1.5	16_2.4	10_3.6	15_2.7	1_3.4	10_2.5	6_2.7	7_1.5	11_1.5	2_1.5	5_1.4
a_n	100	100	100	200	400	100	100	200	400	700	1,300	2,400	100	100	100	100
t_n	100	100	200	400	800	100	200	400	800	1,500	2,800	5,200	100	100	100	100
$W_n\_G_n$	3_120	12_5,460	5_7,510	6_360	5_300	3_620	15_2,060	5_430	6_3,650	6_1,120	12_250	6_260	7_140	11_130	2_130	5_130
s_n	120				1,200							6,240	140	130	130	130
u_n		100	200	400		100	200	400	800	1,500	2,800					
P_n	20	-80	-180	-380	420	320	220	20	-380	-1,080	-2,380	1,460	1,500	1,530	1,560	1,590

Fig. 8 shows some examples. For R6, we are able to reduce the betting count and raise the hit rate, but in this example, the value of R6's $A_{\max}$ becomes larger.



(a)	b	120	$A_{\max}$	1.7E+5	c_b	1,025	(b)	b	1,390	$A_{\max}$	4.9E+5	c_b	2,913	(c)	b	9,900	$A_{\max}$	300	c_b	1,016
R1	r_u	40.49	r_h	41.66	r_r	124.4	R6	r_u	9.337	r_h	9.371	r_r	133.9	N4	r_u	2.854	r_h	2.854	r_r	235.5

Fig. 8. Profit change of Type V (ν=10, ξ=1.5).

Section 6 Increasing the value of coefficient b (Type VI)

When the value of b is small, the input amount can rapidly become large. Thus, increasing the value of b when the uncollected amount is large is an effective strategy.

Let q be the number of candidates for the value of b. In this section, we treat the case of q=6.

Let ${}^1b$, …, ${}^qb$ be the candidates of b, and let b^ be the standard value of b. We predetermine these values. Table 24 is an example for the case of q=6.

Let ${}_1e$, …, ${}_qe$ be the coefficient which moves to next larger ${}^jb$ (j=1,…, q-1). Table 25 is an example corresponding to Table 24.

We determine b as follows:

For j=q to 1 step −1

(16) If $u_{(n-1)} \geqq {}_je \times b$^ then b=${}^jb$, Exit For

Next j

We compute the input amount as follows:

If ($o_n \geqq (b+100) \div 100$ and $u_{(n-1)}>0$)

or ($o_n \leqq \xi$ and $u_{(n-1)} \leqq 0$ and $\sigma_{\nu n}=0$)

(17) then ($a_n = \lceil u_{(n-1)} \div b \rceil \times a_{init}$ If $a_n=0$ then $a_n = a_{init}$)

Else $a_n=0$

If $\sigma_{\nu n}=0$, bet the first popular horse.

Table 24. Configuration of b for Type VI and VII

	${}^1b$	${}^2b$	${}^3b$	${}^4b$	${}^5b$	${}^6b$	b^
Rr Strategy	$R_{(50\%)}$ -100	{0.8× $R_{(50\%)}$ +0.2× $R_{(97.5\%)}$} -100	{0.6× $R_{(50\%)}$ +0.4× $R_{(97.5\%)}$} -100	{0.4× $R_{(50\%)}$ +0.6× $R_{(97.5\%)}$} -100	{0.2× $R_{(50\%)}$ +0.8× $R_{(97.5\%)}$} -100	$R_{(97.5\%)}$ -100	$R_{(50\%)}$ -100
R1	120	162	204	246	288	330	120
R6	1,390.0	1,739.3	2,088.6	2,437.9	2,787.2	3,136.5	1,390.0
Nn Strategy	4,900	5,900	6,900	7,900	8,900	9,900	9,900

Table 25. Configuration of *e* for Type VI

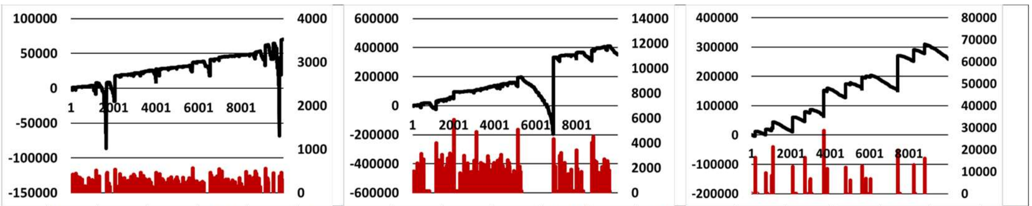
	$_1e$	$_2e$	$_3e$	$_4e$	$_5e$	$_6e$
R*r* Strategy and N*n* Strategy	0	5	10	15	20	25

Table 26 shows the first 16 betting races. Compared with Table 23, we can see that changing the value of *b* results in more races being skipped after the 47th race.

Table 26. First 16 betting races of Type VI

n	15	28	31	32	33	43	44	45	46	47	54	60	68	80	104	160
$g^1{}_n$_$o^1{}_n$	3_1.2	11_1.5	4_3.1	2_3.5	5_3	7_1.5	16_2.4	10_3.6	15_2.7	1_3.4	14_3.4	4_4	16_4	2_1.5	5_1.4	3_1.5
b	120	120	120	120	120	120	120	120	120	162	204	246	288	120	120	120
a_n	100	100	100	200	400	100	100	200	400	500	700	900	1,100	100	100	100
t_n	100	100	200	400	800	100	200	400	800	1,300	2,000	2,900	4,000	100	100	100
W_n_G_n	3_120	12_5,460	5_7,510	6_360	5_300	3_620	15_2,060	5_430	6_3,650	6_1,120	13_370	6_410	16_420	2_130	5_130	3_140
s_n	120				1,200								4,620	130	130	140
u_n		100	200	400		100	200	400	800	1,300	2,000	2,900				
P_n	20	-80	-180	-380	420	320	220	20	-380	-880	-1,580	-2,480	1,040	1,070	1,100	1,140

Fig. 9 shows an example using the same parameters as Tables 24 and 25. By this operation, we can reduce the betting count, and the value of A_{max} becomes smaller for R*r* strategies. For N*n* strategies, it has no effect.



(a)	*b*	-	A_{max}	40,000	c_b	790	(b)	*b*	-	A_{max}	12,500	c_b	2,084	(c)	*b*	-	A_{max}	600	c_b	1,687
R1	r_u	38.48	r_h	39.49	r_r	110.7	R6	r_u	9.836	r_h	9.836	r_r	136.3	N4	r_u	2.726	r_h	2.726	r_r	187.5

Fig. 9. Examples of Type VI (*q*=6, *v*=10).

Tables 27 and 28 summarize the results of Types I to VI. For R*r* strategies, as the Type number increases, both the betting count and the maximum input amount tend to decrease. On the other hand, for N*n* strategies, the maximum input amount is already a small value at Type III, so we can't say with certainty which is best.

Table 27. Results by horse rank

const bet	Type I		Type II		Type III		Type IV		Type V		Type VI	
rank/*b*	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}
r_h/r_r	r_u	r_r	r_u	r_r	r_u	r_r	r_u	r_r	r_u	r_r	r_u	r_r
R1 / 120	10,000	2.0E+8	8,202	3.9E+7	6,920	2.1E+7	3,154	2.1E+7	1,025	1.7E+5	790	40,000
34.4 / 81.4	24.14	140.5	28.151	121.7	27.991	123.5	27.298	116.5	40.487	124.4	38.481	110.7
R2 / 290	10,000	3.0E+7	7,507	6.8E+6	6,464	5.1E+6	4,206	3.8E+6	1,422	2.8E+6	938	48,800
19.4 / 79.7	14.48	122.5	16.198	122.8	15.083	120.6	15.691	119.9	27.355	135.1	25.159	107.9
R3 / 510	10,000	3.1E+6	7,699	5.5E+7	6,346	4.6E+7	4,803	2.7E+7	1,710	4.5E+6	1,015	2.4E+5
12.4 / 81.6	9.33	119.5	10.533	125	10.006	124.7	10.014	124.7	21.111	126	17.142	114.6
R4 / 750	10,000	2.8E+6	7,712	9.1E+7	6,526	5.5E+7	5,360	4.9E+7	2,124	1.4E+7	1,181	20,700
8.6 / 80.8	6.55	115.6	7.326	108.8	6.742	109.3	6.884	107.5	15.913	106.6	17.358	77.2
R5 / 1060	10,000	4.0E+5	7,583	3.1E+5	6,375	2.8E+5	5,519	2.8E+5	2,657	27,300	1,489	1.6E+5
6.3 / 79.1	4.76	118.8	5.288	131.5	4.988	132.1	4.946	132.9	10.801	127.2	9.335	37.6
R6 / 1390	10,000	4.0E+5	7,758	5.6E+5	6,476	4.6E+5	5,833	3.5E+5	2,913	4.9E+5	2,084	12,500
4.9 / 82.4	3.82	135.8	4.253	119.9	3.798	120.5	3.823	121.2	9.337	133.9	9.836	136.3
R7 / 1790	9,914	6.4E+5	7,520	8.3E+5	6,411	6.0E+5	5,895	6.0E+5	3,163	61,400	1,558	4.4E+5
3.6 / 75.2	2.703	113.4	2.952	115	2.885	116.1	2.832	115.2	7.334	125.9	7.06	16.9
R8 / 2220	9,712	5.2E+6	7,404	8.3E+6	6,385	7.3E+6	5,992	7.3E+6	3,435	4.3E+6	1,316	16,000
2.7 / 73.8	2.059	172.4	2.255	128.7	2.161	128.8	2.219	127.9	5.56	120.9	7.294	110.1
R9 / 2645	9,423	2.1E+6	7,409	2.4E+5	6,556	2.0E+5	6,239	1.6E+5	3,885	59,600	1,810	31,800
2.1 / 68.9	1.56	155.5	1.781	179.7	1.571	179.8	1.618	181.9	4.324	179.2	2.817	103.1
R10 / 3430	8,917	7.2E+6	6,880	3.2E+5	5,856	2.3E+5	5,599	2.1E+5	3,702	99,700	1,874	3,300
2 / 82.6	1.558	288.6	1.715	227.2	1.553	227.1	1.535	228.3	3.295	208.8	2.721	121.7
R11 / 3925	8,249	1.3E+6	6,623	1.4E+9	5,886	1.1E+9	5,693	1.1E+9	3,975	2.8E+8	1,534	1.3E+5
1.5 / 72	1.018	117.8	1.132	171.3	1.138	171.3	1.141	171.3	2.238	171.3	1.173	97.9
R12 / 4580	7,419	3.2E+5	6,100	2.3E+6	5,611	1.8E+6	5,497	1.7E+6	3,603	8.1E+5	2,370	7,800
1.2 / 73.4	0.916	115.1	1.032	127.4	0.837	127.2	0.855	126.9	1.942	129.3	1.898	135.8
R13 / 5900	6,492	1.9E+5	5,356	2.0E+5	4,778	3.0E+5	4,725	3.0E+5	3,199	28,500	1,163	700
1 / 88.4	0.754	197	0.821	218	0.69	219.8	0.719	219.9	1.875	210.4	2.493	96.7
R14 / 8220	5,469	88,700	4,302	24,400	3,753	18,200	3,709	18,000	2,509	5,600	1,776	5,100
0.6 / 61	0.493	111.3	0.534	118.8	0.479	120.1	0.485	120.2	1.514	116.6	0.957	97.8
R15 / 9265	4,388	2.1E+6	3,604	1.8E+8	3,477	1.5E+8	3,473	1.5E+8	2,926	1.3E+7	1,581	82,400
0.3 / 33.5	0.182	114	0.166	163	0.143	163	0.143	163	0.307	163	0.063	1.3
R16 / 15490	3,007	12,500	2,415	22,400	2,089	16,700	2,089	16,700	1,605	13,400	1,324	1,000
0.4 / 80.2	0.299	180.5	0.372	240.6	0.335	240.7	0.335	240.7	0.809	242.7	0.981	156.1

Table 28. Results by horse number

const bet	Type I		Type II		Type III		Type IV		Type V		Type VI	
No/ b	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}	C_b	A_{max}
r_h/r_r	r_u	r_r	r_u	r_r	r_u	r_r	r_u	r_r	r_u	r_r	r_u	r_r
N1 / 9900	10,000	1.2E+8	2,164	2,000	1,064	900	1,063	900	1,027	800	1,381	2,300
7.7 / 70.1	0.13	144.4	0.646	164.4	0.281	165.8	0.282	166.2	0.876	165.2	1.303	179.1
N2 / 9900	10,000	20,100	3,127	3,400	1,162	1,500	1,158	1,500	1,099	1,500	1,604	3,400
8.1 / 79.1	0.23	106	0.639	123.9	0.602	135.1	0.604	135.5	1.546	137.4	1.807	128.3
N3 / 9900	10,000	2.7E+8	2,471	9,500	1,161	4,300	1,158	4,300	1,103	3,600	1,399	8,700
7.5 / 71.1	0.19	134.5	0.647	83	0.602	86.9	0.604	87.3	1.087	87	1.429	62.5
N4 / 9900	10,000	63,000	3,376	800	1,138	300	1,131	300	1,016	300	1,687	600
8.4 / 86.2	0.41	150.2	0.71	156.9	0.966	201.3	0.972	202.6	2.854	235.5	2.726	187.5
N5 / 9900	10,000	16,900	2,040	1.7E+5	1,094	1.1E+5	1,092	1.1E+5	1,063	87,000	1,233	2.1E+5
8.5 / 84.1	0.31	109	0.735	113	0.274	113	0.274	113.1	0.47	113.1	0.486	113
N6 / 9900	10,000	40,400	2,020	96,200	1,112	57,600	1,112	57,600	1,070	43,500	1,243	1.1E+5
8 / 83.4	0.32	85.9	0.594	107.3	0.269	107.8	0.269	107.8	1.028	107.4	1.367	107.8
N7 / 9900	9,914	1.9E+5	2,219	4,500	1,117	2,100	1,115	2,100	1,054	1,800	1,379	4,000
7.8 / 75.5	0.171	101.3	0.721	213.6	0.537	227.6	0.538	227.9	1.233	223	1.015	194.8
N8 / 9900	9,712	3.0E+8	1,373	9.5E+5	1,116	4.1E+5	1,115	4.1E+5	1,073	3.0E+5	1,184	6.9E+5
7.5 / 68.6	0.051	178.5	0.364	380.6	0.089	380.6	0.089	380.6	0.279	380.5	0.253	380.5
N9 / 9900	9,423	2.0E+10	1,428	5.3E+5	1,114	2.6E+5	1,114	2.6E+5	1,070	1.8E+5	1,181	3.0E+5
6.3 / 61.5	0.042	45	0.14	0.5	0.089	0.4	0.089	0.4	0.186	0.6	0.508	0.8
N10 / 9900	8,917	2.0E+6	2,242	47,100	1,087	19,900	1,082	19,900	1,029	16,100	1,314	39,600
6.5 / 68.7	0.19	114.2	0.758	165.5	0.551	168.2	0.554	168	1.263	167.6	0.989	166.5
N11 / 9900	8,249	1.2E+5	1,876	9,500	1,056	4,000	1,053	3,900	1,021	3,500	1,342	4,300
6.1 / 63.6	0.157	110.1	0.426	148.4	0.378	154.2	0.379	153.6	0.587	153.1	0.968	154.9
N12 / 9900	7,419	7,200	1,240	54,800	917	23,300	916	23,100	885	19,500	1,116	24,500
5.3 / 59.1	0.148	90.2	0.241	12.7	0.109	13.4	0.109	13.6	0.225	13.6	0.627	20.7
N13 / 9900	6,492	2.4E+6	2,180	2,500	886	1,200	885	1,200	761	1,200	1,016	2,700
4.1 / 51.9	0.508	101.5	0.917	122.8	0.79	138.3	0.79	138.4	3.942	149.6	4.429	143.4
N14 / 9900	5,469	34,400	1,308	3,700	775	1,800	775	1,800	717	1,200	896	1,200
3.1 / 34.8	0.109	100.1	0.458	98.6	0.258	96.7	0.258	96.7	0.278	95.7	2.008	69.6
N15 / 9900	4,388	3,100	1,224	2,800	625	1,500	624	1,500	532	600	677	800
2.7 / 33.6	0.205	91.2	0.735	47.6	0.32	35.8	0.32	35.9	1.691	68.5	1.624	77.7
N16 / 9900	3,007	8,200	1,026	500	453	200	451	200	333	200	498	500
1.7 / 21.7	0.133	99	0.682	125.6	0.662	132.3	0.665	132.7	3.903	182.6	2.61	152.1

Section 7 Switching to less popular horses (Type VII)

We need not focus only on one horse. Actually, less popular horses can accumulate greater uncollected amounts than more popular horses because the value of b is larger for less popular horses. So, the strategy of switching to a less popular horse when the uncollected amount is large may be effective.

Let R^{start} be the starting rank while the uncollected amount is small, let R^{max} be the maximum rank when the uncollected amount is large, and let ${}^{1}b_r$, …, ${}^{q}b_r$ be candidate b for rank r. We predetermine these values. Table 24 (replace ${}^{1}b$ as ${}^{1}b_r$ for each rank r) is an example for the case of q=6.

Let ${}_{1}f_r$, …, ${}_{q}f_r$ be the coefficient which moves to the next larger ${}^{j}b_r$ (j=1,…, q-1). Table 29 is an example corresponding to Table 24.

Table 29. Configuration of e for Type VII

Rank	${}_{1}f_r$	${}_{2}f_r$	${}_{3}f_r$	${}_{4}f_r$	${}_{5}f_r$	${}_{6}f_r$
1	0	1	2	3	4	5
2	6	7	8	9	10	11
(rank)	(rank-1)×6	(rank-1)×6 +1	(rank-1)×6 +2	(rank-1)×6 +3	(rank-1)×6 +4	(rank-1)×6 +5

We replace STEP6 as follows.

STEP6* Choose the horse rank h_{rn} and compute the value of a_n as follows.

First, decide on the horse rank:

Initialize Exit_flag=False

For $k=R^{\mathrm{max}}$ to R^{start} step -1

For $j=q$ to 1 step -1

(18) If $u_{(n-1)} \geqq {}_{j}f_k \times b$^ then $b={}^{j}b_k$, $h_{rn}=k$, Exit_flag=True, Exit For

Next j

If Exit_flag=True then Exit For

Next k

Next, compute the input amount:

If ($o_n \geqq (b+100) \div 100$ and $u_{(n-1)} > 0$)

or ($o_n \leqq \xi$ and $u_{(n-1)} \leqq 0$ and $\sigma_{vn}=0$)

(19) then ($a_n = \lceil u_{(n-1)} \div b \rceil \times a_{init}$ If $a_n=0$ then $a_n = a_{init}$)

Else $a_n=0$

If $\sigma_{vn}=0$, bet the first popular horse, else bet h_{rn}.

Following the decision of h_{rn}, compute a_n using b (set by ${}_{hrn}b$).

Table 30 shows 16 betting races (the 501st to 763rd races) for the case of $R^{max}=3$, using the configurations from Tables 24 and 29. We can see that, as the wagered horses continue to lose, the rank of the wagered horse changes from being the most popular to the second most popular and then to the third most popular.

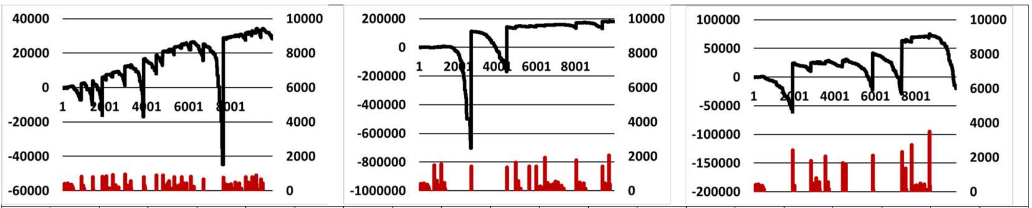
Table 30. Example of betting races of Type VII

c_b	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
n	501	502	503	504	528	529	532	540	548	551	587	604	682	724	755	763
$g^1{}_n$_$o^1{}_n$	7_1.4	3_2.5	12_2.7	1_3.7	15_4.7	15_4.4	4_2.2	3_1.6	3_1.6	9_1.9	4_1.8	3_1.8	12_2.3	7_1.7	4_2.1	6_1.9
$g^2{}_n$_$o^2{}_n$	4_7.4	14_2.9	16_4.5	11_4.3	8_4.8	13_4.5	5_7.6	8_7.7	4_3.3	4_2.2	3_4.6	7_11	11_2.6	10_4.3	14_3.9	12_7
$g^3{}_n$_$o^3{}_n$	5_8.6	11_6.2	7_6.8	2_4.7	1_6.8	5_6.8	8_8.7	5_8.7	9_12.4	1_17.2	13_17.7	14_12.5	2_13.2	12_13.4	13_13.5	15_13.4
h_{rn}_b	1_120	1_120	1_162	1_246	1_330	2_290	2_506	2_650	3_762	3_1014	3_1140	3_1140	3_1140	3_1140	3_1140	3_1140
a_n	100	100	200	200	200	300	300	300	300	200	200	300	300	300	300	400
t_n	100	200	400	600	800	1,100	1,400	1,700	2,000	2,200	2,400	2,700	3,000	3,300	3,600	4,000
W_n_G_n	8_2,000	11_580	7_760	11_440	11_1,410	11_1,730	10_3,240	6_2,220	3_150	4_240	4_180	3_180	11_270	7_170	14_350	12_720
s_n																
u_n	100	200	400	600	800	1,100	1,400	1,700	2,000	2,200	2,400	2,700	3,000	3,300	3,600	4,000
P_n	740	640	440	240	40	-260	-560	-860	-1,160	-1,360	-1,560	-1,860	-2,160	-2,460	-2,760	-3,160

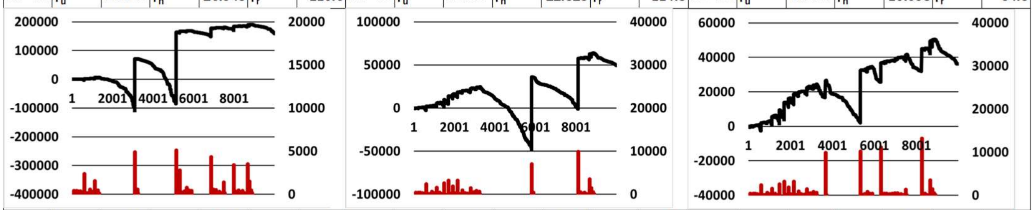
Table 31 and Fig. 10 show examples for the case of $R^{start}=1$. The parameters are set by using Tables 24 and 29. Similarly, Table 32 and Fig. 11 show examples for cases where $R^{start}>1$. There are some cases where the results appear to be excellent. However, if R^{max} changes by 1, the results change significantly. The positive results may be just a coincidence. It's unknown if these ranks would continue to perform well in the future, so I cannot recommend this with confidence.

Table 31. Results of Type VII for the case R^{start}=1

R^{start}～R^{max}	C_b	A_{max}	R^{start}～R^{max}	C_b	A_{max}	R^{start}～R^{max}	C_b	A_{max}	R^{start}～R^{max}	C_b	A_{max}
	r_u	r_r		r_u	r_r		r_u	r_r		r_u	r_r
R1～R2	553	11,000	R1～R6	692	5,000	R1～R10	814	300	R1～R14	974	4,800
	26.22	110.6		17.774	141.4		21.621	125.6		9.24	8.3
R1～R3	551	62,200	R1～R7	806	58,500	R1～R11	936	2,600	R1～R15	1,041	3,800
	22.323	114.8		16.129	87.7		10.683	47.4		8.645	9.1
R1～R4	548	5,300	R1～R8	688	1,200	R1～R12	941	1,200	R1～R16	521	300
	16.058	94.3		16.715	126.8		11.052	123.9		22.072	147.7
R1～R5	717	10,300	R1～R9	904	2,300	R1～R13	523	300			
	17.852	67.4		17.92	139.2		22.753	136.1			



(a)	b	-	A_{max}	11,000	c_b	553	(b)	b	120	A_{max}	62,200	c_b	551	(c)	b	120	A_{max}	5,300	c_b	548
R1～R2	r_u	26.220	r_h	26.943	r_r	110.6	R1～R3	r_u	22.323	r_h	22.323	r_r	114.8	R1～R4	r_u	16.058	r_h	16.058	r_r	94.3

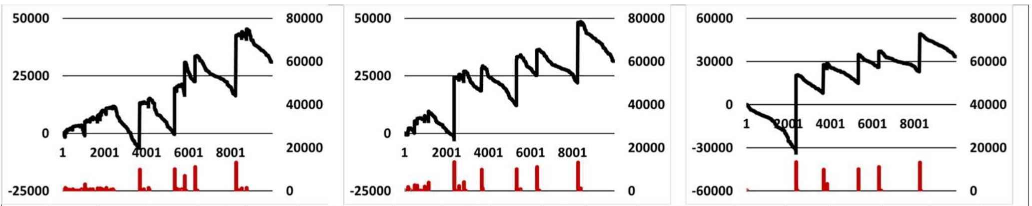


(d)	b	120	A_{max}	5,000	c_b	692	(e)	b	120	A_{max}	1,200	c_b	688	(f)	b	120	A_{max}	300	c_b	814
R1～R6	r_u	17.77	r_h	17.92	r_r	141.4	R1～R8	r_u	16.72	r_h	16.72	r_r	126.8	R1～R10	r_u	21.62	r_h	21.74	r_r	125.6

Fig. 10. Examples of Type VII for the case R^{start}=1.

Table 32. Results of Type VII for cases where R^{start}>1

R^{start}～R^{max}	C_b	A_{max}	R^{start}～R^{max}	C_b	A_{max}	R^{start}～R^{max}	C_b	A_{max}	R^{start}～R^{max}	C_b	A_{max}	Rank	C_b	A_{max}
	r_u	r_r		r_u	r_r		r_u	r_r		r_u	r_r		r_u	r_r
R2～R3	572	18,700												
	18.356	120.178												
R2～R4	576	4,500	R3～R4	481	6,000									
	19.097	81.849		8.939	76.063									
R2～R5	745	10,300	R3～R5	706	15,900	R4～R5	712	30,800						
	10.872	84.574		9.773	57.57		7.865	32.93						
R2～R6	727	4,200	R3～R6	687	4,300	R4～R6	660	2,800	R5～R6	648	2,900			
	14.167	133.865		9.17	139.256		6.666	141.26		6.481	143.8			
R2～R7	792	69,700	R3～R7	809	83,300	R4～R7	796	124,500	R5～R7	824	47,900	R6～R7	794	87,600
	9.722	54.228		7.663	73.61		4.02	79.144		4.126	55.976		5.919	70.995
R2～R8	652	1,700	R3～R8	626	3,200	R4～R8	594	3,200	R5～R8	612	9,500	R6～R8	559	9,000
	12.423	119.66		9.265	116.374		7.407	112.974		3.758	110.381		2.504	109.067
R2～R9	936	4,600	R3～R9	941	3,700	R4～R9	948	3,300	R5～R9	1,032	2,700	R6～R9	1,007	3,100
	8.974	116.433		4.675	115.115		4.43	118.113		3.972	124.517		2.383	114.886
R2～R10	815	300	R3～R10	782	200	R4～R10	806	200	R5～R10	802	400	R6～R10	805	400
	12.515	126.209		8.184	129.033		6.203	132.794		2.244	123.605		2.36	132.272
R2～R11	961	2,700	R3～R11	979	5,400	R4～R11	978	6,200	R5～R11	1,034	21,000	R6～R11	1,053	15,200
	7.804	65.464		4.902	85.321		2.351	75.504		1.257	96.771		1.329	93.896
R2～R12	974	800	R3～R12	979	1,300	R4～R12	945	500	R5～R12	1,048	1,200	R6～R12	1,090	1,200
	8.11	119.815		5.822	134.938		4.973	140.383		1.24	129.548		1.926	132.876
R2～R13	432	300	R3～R13	423	200	R4～R13	342	200	R5～R13	255	200	R6～R13	291	100
	16.898	126.836		9.692	161.425		7.602	134.767		1.176	104.448		1.718	129.793
R2～R14	996	2,000	R3～R14	1,031	3,100	R4～R14	1,026	3,100	R5～R14	1,064	2,900	R6～R14	1,076	3,000
	7.831	29.044		3.006	18.444		2.046	17		0.375	26.75		0.371	23.708
R2～R15	1,078	6,300	R3～R15	1,123	9,800	R4～R15	1,116	11,400	R5～R15	1,180	11,000	R6～R15	1,189	9,200
	5.658	2.944		2.671	1.407		1.792	0.851		0.423	1.136		1.766	2.749
R2～R16	466	300	R3～R16	386	200	R4～R16	336	200	R5～R16	320	200	R6～R16	313	100
	17.167	143.52		9.844	152		6.845	159.749		3.437	169.487		2.875	170.063



(a)	*b*	-	A_{max}	300	C_b	815	(b)	*b*	120	A_{max}	200	C_b	806	(c)	*b*	120	A_{max}	400	C_b	805
R2～R10	r_u	12.515	r_h	12.515	r_r	126.2	R4～R10	r_u	6.203	r_h	6.203	r_r	132.8	R6～R10	r_u	2.360	r_h	2.36	r_r	132.3

Fig. 11. Examples of Type VII for cases where R^{start}>1.

Section 8 Results of more data

In the previous sections, we designed several models using data from 10,000 races, referred to as data0. There were some good results. However, we don't yet know whether it is safe to use the same models in future races. To address this, I prepared more data using the results of 100,000 (10,000×10) races, referred to as data1 to data10. I collected the results using the same parameters as those applied to data0. In Section 2, to address the safety of constant betting models, I used the ratio of the minimum recovery rates of data1 to data10 to the recovery rate of data0, I_Y as shown in Tables 13 and 14. For constant betting with the higher rank horses, the recovery rates did not drop significantly. For doubling, I used the ratio of the maximum input amount of data1 to data10 to the maximum input amount of data0 as an evaluation indicator, using the notation I_Z for this value. This confirms whether a model that appears safe in data0 is really safe over 100,000 races.

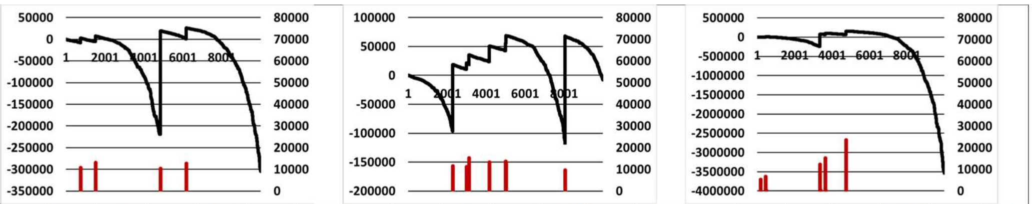
Table 33 shows the results of the N*n* strategy applied to the data from 100,000 races. The values of the parameters for data1 to data10 are the same as those for data0. In Table 28, it seems that N4 has a large advantage in Types III, IV, and V (the value of A_{max} is 300 for these types). However, in Table 33, the value of A_{max} for N4 exceeds 300. For Type V, the value of I_Z reaches 54.7. N1 looked relatively good with data0 but had similarly terrible results, with I_Z reaching 113.6. For N16, Z seems to be a relatively good value, yet there are some examples of the recovery rates declining. In the first place, the betting counts for N16 were small, so we cannot make a sufficient evaluation.

From these results, we can say that N4 has an advantage. However, we cannot assess the safety of the individual horse numbers or the strategies used. In actual operation, we would need to be prepared for the input amount to grow to 100 to 1,000 times the value of A_{max} seen in the historical data. It may even exceed that.

Fig. 12 shows some examples of bad results.

Table 33. Results of N*n* strategy of Type III to VI for small $A_{\max}$ in data0

Type		Type III						Type IV					
N*n*		N1		N4		N16		N1		N4		N16	
		$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r
data0		900	165.9	300	201.3	200	132.3	900	166.2	300	202.6	200	132.7
Test data	data1	102,200	127.1	2,500	104.6	400	69.9	102,200	127.1	2,500	105.0	400	69.9
	data2	3,100	75.9	3,400	48.8	2,200	117.8	3,100	76.2	3,400	48.7	2,200	117.8
	data3	96,200	234.3	3,300	100.2	500	73.2	96,200	234.2	3,300	100.1	500	73.1
	data4	1,100	132.9	4,300	126.5	2,900	108.2	1,100	134.1	4,300	125.0	2,900	108.1
	data5	7,300	118.6	500	126.5	2,500	6.1	7,300	118.5	500	127.7	2,500	6.0
	data6	3,000	170.7	4,000	226.9	700	130.7	3,000	171.0	4,000	228.8	700	130.7
	data7	18,100	181.9	1,900	98.0	900	295.6	18,000	182.6	1,900	99.7	900	295.6
	data8	1,900	208.0	800	140.0	3,500	154.0	1,800	205.5	800	135.8	3,500	154.0
	data9	39,300	113.5	1,900	131.2	1,300	141.6	38,900	113.4	1,900	131.4	1,300	141.5
	data10	6,300	111.4	5,900	139.1	500	136.5	6,300	111.4	5,900	139.1	500	136.4
max$A_{\max}$	min_r_r	102,200	75.9	5,900	48.8	3,500	6.1	102,200	76.2	5,900	48.7	3,500	6.0
avg$A_{\max}$	avg_r_r	27,850	147.4	2,850	124.2	1,540	123.4	27,790	147.4	2,850	124.1	1,540	123.3
I_Z			113.6		19.7		17.5		113.6		19.7		17.5
Type		Type V						Type VI					
N*n*		N1		N4		N16		N1		N4		N16	
		$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r	$A_{\max}$	r_r
data0		800	165.3	300	235.5	200	182.6	2,300	179.1	600	187.5	500	152.1
Test data	data1	81,900	126.8	16,400	16.6	400	109.1	199,800	126.8	36,900	11.6	500	76.8
	data2	2,800	75.4	2,700	53.1	2,000	116.2	5,300	62.9	7,000	47.7	800	142.3
	data3	74,100	234.3	2,900	101.0	500	82.0	177,100	234.1	6,300	96.9	1,100	77.5
	data4	1,000	123.1	3,600	125.4	1,700	108.2	2,400	115.5	8,300	123.7	3,500	106.8
	data5	6,100	126.1	400	136.9	1,400	11.3	10,500	126.2	1,100	117.8	3,100	5.2
	data6	2,600	170.7	3,100	229.6	700	140.2	6,600	165.7	7,900	223.6	1,200	147.0
	data7	14,300	183.3	1,600	96.6	900	316.8	34,800	181.4	2,100	102.1	2,100	296.3
	data8	1,700	216.7	700	137.6	2,100	155.8	3,800	243.5	1,800	134.7	4,700	158.7
	data9	29,100	113.8	1,600	130.1	500	136.2	64,200	114.5	1,900	124.7	1,400	143.5
	data10	5,000	115.2	4,600	137.1	3,200	0.1	9,800	111.0	5,200	129.7	2,200	9.3
max$A_{\max}$	min_r_r	81,900	75.4	16,400	16.6	3,200	0.1	199,800	62.9	36,900	11.6	4,700	5.2
avg$A_{\max}$	avg_r_r	21,860	148.5	3,760	116.4	1,340	117.6	51,430	148.1	7,850	111.2	2,060	116.4
I_Z			102.4		54.7		16.0		86.9		61.5		9.4



(a)	b	9,900	$A_{\max}$	3,400	c_b	1,088	(b)	b	9,900	$A_{\max}$	1,900	c_b	1,196	(c)	b	-	$A_{\max}$	36,900	c_b	1,315
N4	r_u	0.367	r_h	0.367	r_r	48.75	N4	r_u	0.501	r_h	0.585	r_r	98.01	N4	r_u	0.836	r_h	0.836	r_r	11.59
Type III	data2						Type III	data7						Type VI	data1					

Fig. 12. Examples of bad results for N*n* strategy.

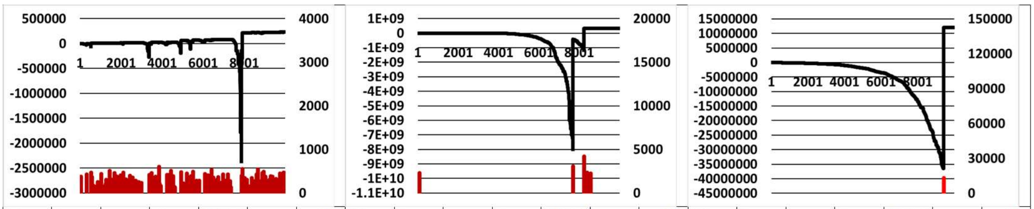
Table 34 and Fig. 13 show the results of Type VI for the R*r* strategy. In Table 27, it seems that some R*r* strategies have good results. However, in Table 34, we can see values of $A_{\max}$ exceeding 100,000. From these results, it's clear that even if we see good results in the historical data, the fact that there are significant risks is undeniable. In the R6 strategy, the value of $A_{\max}$ from data0 is relatively small at 12,500, and this makes it appealing as a method to try out. However, in the first 10,000 races, $A_{\max}$ reaches 2.5E+8, which is 20,289 times that of data0. This shows how dangerous this model is. Although it did not meet the author's expectations, I must emphasize this result.

Table 34. Results of R*r* strategy of Type VI

R*r*		R1		R2		R3		R4		R5		R6	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		40,000	110.7	48,800	108.0	2.4E+5	114.6	20,700	77.3	1.6E+5	37.7	12,500	136.3
Test data	data1	40,000	120.4	84,200	110.6	10,500	126.5	86,300	112.3	49,500	137.8	2.5E+8	103.8
	data2	17,300	116.6	9.6E+5	101.6	18,900	121.5	24,600	111.7	5,000	127.2	1.2E+5	108.5
	data3	7.4E+5	104.8	54,800	112.7	5.6E+7	108.9	15,600	124.8	1.7E+5	106.8	3,800	143.7
	data4	30,700	114.5	47,500	109.8	13,500	114.2	3.8E+5	103.2	6,500	131.8	2.1E+5	105.7
	data5	1.1E+5	109.7	2.6E+5	102.5	10,500	96.9	1.1E+5	140.2	73,500	106.7	6.1E+5	119.9
	data6	2.6E+5	115.5	17,400	113.3	3.6E+5	103.4	5.1E+5	120.0	1.2E+5	142.9	3.0E+6	160.9
	data7	23,600	117.4	2.8E+6	151.0	85,900	148.2	17,500	112.0	8.6E+5	110.5	43,900	99.1
	data8	2.0E+5	119.1	1.6E+5	114.4	12,400	120.3	86,300	112.5	41,500	125.5	30,100	112.9
	data9	40,000	111.4	41,200	118.1	2.5E+6	100.6	8,800	109.4	30,500	140.6	2.2E+6	101.8
	data10	52,200	110.2	1.9E+6	116.0	37,100	117.5	4.4E+6	137.3	22,400	128.6	46,700	124.2
maxA_{max}	min_r_r	7.4E+5	104.8	2.8E+6	101.6	5.6E+7	96.9	4.4E+6	103.2	8.6E+5	106.7	2.5E+8	99.1
avgA_{max}	avg_r_r	1.4E+5	114.0	5.8E+5	115.0	5.4E+6	115.8	5.2E+5	118.3	1.4E+5	125.8	2.4E+7	118.1
I_Z			18.4		57.6		232.1		214.8		5.4		20,289.6

R*r*		R7		R8		R9		R10		R11	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		4.4E+5	17.0	16,000	110.1	31,800	103.1	3,300	121.7	1.3E+5	98.0
Test data	data1	6.6E+7	0.0	17,600	106.9	37,400	106.1	3.7E+5	132.6	2.6E+5	101.8
	data2	5,700	129.7	5,800	131.9	28,700	201.1	4,000	118.1	9,700	118.7
	data3	5,500	132.4	1.5E+5	109.5	8,200	170.7	14,300	118.6	1.2E+5	132.7
	data4	19,500	146.2	24,100	136.0	2.5E+5	108.1	10,600	132.6	46,600	106.0
	data5	81,800	111.7	43,000	100.5	34,300	106.4	5,200	142.6	6,200	129.6
	data6	9,900	109.0	57,900	98.3	41,500	118.9	4,700	91.8	44,500	100.7
	data7	6,600	127.6	16,500	156.1	1.5E+6	1.2	2,300	157.6	74,900	101.0
	data8	4.8E+5	107.3	17,300	33.4	11,700	125.1	3.1E+5	117.8	12,200	128.5
	data9	4,200	142.1	57,000	112.1	6,300	170.8	10,500	125.9	2.8E+6	126.2
	data10	9,900	114.6	14,100	142.4	10,500	87.7	3,200	118.4	72,800	139.0
maxA_{max}	min_r_r	6.6E+7	0.0	1.5E+5	33.4	1.5E+6	1.2	3.7E+5	91.8	2.8E+6	100.7
avgA_{max}	avg_r_r	6.1E+6	112.1	37,755	112.7	1.8E+5	119.6	66,864	125.6	3.3E+5	118.4
I_Z			149.8		9.1		47.8		111.4		21.6

R*r*		R12		R13		R14		R15		R16	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		7,800	135.8	700	96.7	5,100	97.9	82,400	1.3	1,000	156.1
Test data	data1	31,500	12.8	800	6.3	3,500	85.6	1.4E+5	83.6	800	100.2
	data2	1,900	142.4	800	4.6	15,000	85.0	5,500	78.2	1,000	66.5
	data3	48,100	108.5	700	20.5	41,000	6.9	12,700	103.0	1,300	0.0
	data4	5,700	153.0	800	12.4	54,500	5.1	11,100	105.9	800	140.8
	data5	19,500	125.9	800	39.2	2.8E+5	112.4	10,800	107.0	1,300	0.1
	data6	7,400	143.9	700	127.8	1.0E+5	1.7	65,700	118.5	1,300	0.0
	data7	11,500	122.1	800	0.0	1,900	136.5	6,500	111.1	1,300	4.5
	data8	34,200	113.2	700	51.8	1,000	151.7	3,000	78.2	1,000	111.9
	data9	8,100	165.9	700	74.2	8,900	122.6	30,100	114.9	900	65.5
	data10	29,800	131.6	800	0.0	42,000	104.6	9,100	105.2	1,300	0.1
maxA_{max}	min_r_r	48,100	12.8	800	0.0	2.8E+5	1.7	1.4E+5	78.2	1,300	0.0
avgA_{max}	avg_r_r	18,682	121.9	754.5	33.7	50,191	81.2	33,945	100.6	1,091	48.9
I_Z			6.2		1.1		54.1		1.7		1.3



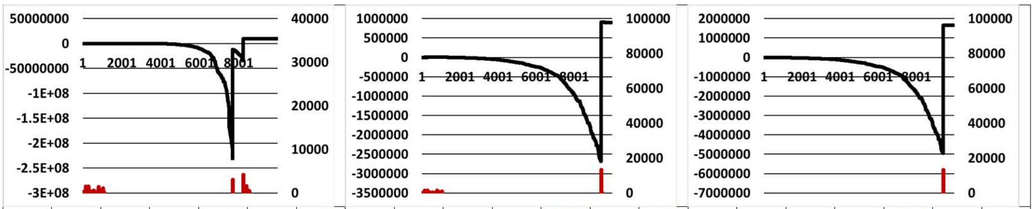
(a)	b	-	A_{max}	7.4E+5	c_b	835	(b)	b	-	A_{max}	2.5E+8	c_b	691	(c)	b	-	A_{max}	3.7E+5	c_b	934
R1	r_u	31.98	r_h	32.69	r_r	104.8	R6	r_u	2.749	r_h	2.894	r_r	103.8	R10	r_u	0.107	r_h	0.107	r_r	132.6
Type VI	data3						Type VI	data1						Type VI	data1					

Fig. 13. Examples of bad results from Table 34.

Table 35 and Fig. 14 show the results of Type VII. It seems to have lower risk than that shown in Table 34. However, we can see that the R3～R10 strategy has a value for A_{max} of 300 with data0, but has a value for A_{max} of 43,600 with data1. I cannot recommend these strategies with confidence.

Table 35. Results of Type VII for small A_{max} in data0

R^{start}～R^{max}		R1～R2		R1～R3		R1～R4		R1～R6		R1～R8		R1～R10		R1～R12	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		11,000	110.6	62,200	114.8	5,300	94.3	5,000	141.4	1,200	126.8	300	125.6	1,200	123.9
Test data	data1	2.6E+5	104.0	9,000	123.9	19,800	111.4	7.3E+6	103.8	2,000	110.3	27,100	132.8	5,500	16.8
	data2	1.7E+5	103.1	10,600	120.7	9,400	112.2	5,700	110.4	1,900	139.6	1,300	128.0	1,000	143.5
	data3	22,500	113.3	1.5E+7	108.8	2,800	127.3	14,600	135.7	18,900	107.4	4,800	123.8	8,500	107.0
	data4	12,700	109.5	3,000	119	65,500	107.0	42,500	105.0	2,200	129.1	1,600	128.5	500	132.8
	data5	14,700	112.9	3,600	95.6	73,400	141.1	2.5E+5	120.4	3,800	98.4	1,000	162.8	8,900	130.4
	data6	4,700	120.6	87,100	102.9	1.9E+5	131.8	3.1E+5	160.7	9,600	98.7	700	96.6	1,100	160.6
	data7	6.4E+5	150.5	31,800	138.9	5,300	120.0	5,500	128.7	1,400	172.2	400	152.3	1,900	128.7
	data8	31,800	114.6	2,500	121.8	19,800	114.0	3,900	151.2	3,000	44.9	26,300	118.3	6,300	116.0
	data9	26,000	122.1	9.2E+5	100.8	1,400	110.2	3.2E+5	101.9	19,800	111.1	2,300	120.0	1,000	138.3
	data10	3.5E+5	116.2	7,600	114.2	8.7E+5	137.2	12,100	122.4	2,900	137.3	900	135.0	3,600	122.3
maxA_{max}	min_r_r	6.4E+5	103.1	1.5E+7	95.6	8.7E+5	107.0	7.3E+6	101.9	19,800	44.9	27,100	96.6	8,900	16.8
avgA_{max}	avg_r_r	1.5E+5	116.7	1.6E+6	114.7	1.3E+5	121.2	8.3E+5	124.0	6,550	114.9	6,640	129.8	3,830	119.6
I_Z			58.6		236.5		163.3		1,460.3		16.5		90.3		7.4
R^{start}～R^{max}		R2～R3		R2～R10		R3～R10		R4～R10		R5～R10		R6～R10			
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r		
data0		18,700	120.1	300	126.2	200	129.0	200	132.7	400	123.6	400	132.2		
Test data	data1	4,900	128.4	39,300	133.1	50,000	132.9	20,500	133.3	46,100	132.8	43,900	132.9		
	data2	17,200	120.2	1,300	131.5	1,200	130.0	800	131.5	800	133.2	1,000	147.9		
	data3	1.1E+7	108.8	4,400	117.3	4,800	119.9	3,600	117.3	3,500	120.5	3,500	120.2		
	data4	3,200	118.4	1,400	140.9	1,400	134.9	1,300	134.9	1,300	134.5	1,300	135.1		
	data5	2,100	125.6	1,000	155.1	600	151.8	1,100	149.9	900	144.3	900	147.6		
	data6	51,400	104.7	700	89.7	500	103.7	800	107.7	600	92.8	700	103.2		
	data7	24,100	131.3	400	187.6	400	200.0	300	181.6	300	183.0	300	207.3		
	data8	2,300	128.6	21,300	117.7	34,800	118.1	37,700	117.7	36,600	117.8	34,100	117.8		
	data9	7.6E+5	100.7	1,000	128.5	1,800	129.0	1,100	138.3	1,600	123.5	1,500	128.3		
	data10	4,100	116.5	400	149.3	700	119.4	700	157.7	600	146.5	600	141.0		
maxA_{max}	min_r_r	1.1E+7	100.7	39,300	89.7	50,000	103.7	37,700	107.7	46,100	92.8	43,900	103.2		
avgA_{max}	avg_r_r	1.2E+6	118.3	7,120	135.1	9,620	134.0	6,790	137.0	9,230	132.9	8,780	138.1		
I_Z			596.9		131.0		250.0		188.5		115.3		109.8		



(a)	b	-	A_{max}	7.3E+6	c_b	614	(b)	b	-	A_{max}	27,100	c_b	742	(c)	b	-	A_{max}	50,000	c_b	728
R1～6	r_u	7.817	r_h	8.143	r_r	103.8	R1～10	r_u	5.39	r_h	5.525	r_r	132.8	R3～10	r_u	0.137	r_h	0.137	r_r	132.9
Type VII	data1						Type VII	data1						Type VII	data1					

Fig. 14. Examples of bad results from Table 35.

Section 9 The difficulty of recovery

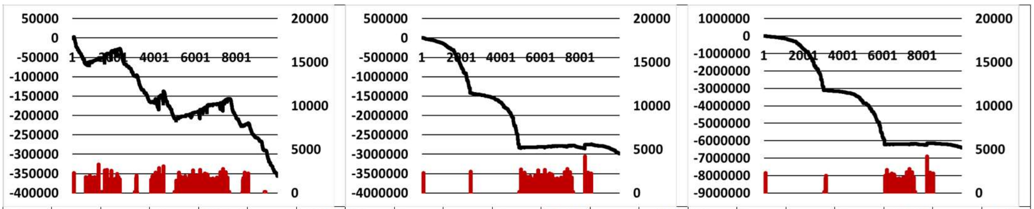
In previous sections, we saw that there is still a risk of generating a large uncollected amount. Is there any way to avoid this situation? In this section, I will introduce two simple examples of failure.

9.1 Clear the uncollected amount when it reaches a predetermined value

The first method is based on the clearing the uncollected amount when it reaches a predetermined value. Intuitively, we have the following expectation: Even if we encounter failures occasionally, if we can continue to have successes, won't we eventually make a profit?

Let AG be a parameter that controls when to abandon increasing the input amount and when to reset the uncollected amount to zero. That is, we clear the uncollected amount when it reaches a value of $b\times$AG.

Fig. 15 shows some examples using Type VI. Table 34 contains an example of a negative outcome with data1 for the R6 strategy. In Type VI, b changes by the uncollected amount, so the relationship between A_{max} and AG is difficult to understand. By this operation, if AG happens to be an effective value, it may be successful, but this is hard to predict. A model that fails once will fail a second time before recovering. While the loss in the case of failure is large, the profit from each success is too small. We won't be able to recover unless we have a great number of successes in a row.



(a)	b	-	A_{max}	2.2E+3	c_b	1,914	(b)	b	-	A_{max}	43,900	c_b	1,316	(c)	b	-	A_{max}	96,200	c_b	1,163
R6	r_u	0.261	r_h	8.568	r_r	54.24	R6	r_u	0.379	r_h	6.914	r_r	9.758	R6	r_u	0.429	r_h	5.503	r_r	4.002
AG=50	data1						AG=1,000	data1						AG=2,200	data1					

Fig. 15. Examples of clearing uncollected amount of Type VI.

9.2 Partition a large uncollected amount

The second method is to divide the uncollected amount into partitions. We take out loans to purchase cars and houses. We use installment payments when we incur a large debt. This is a similar idea. What would happen if we divide the uncollected amount when it becomes large?

Let d_n be the divided uncollected amount. Let d^{sta} be the starting value of the divided uncollected amount. Let d^{back} be the threshold set to the minimum value for the uncollected amount while d_n remains (a threshold for when the uncollected amount falls below this value). We use predetermined values for d^{sta} and d^{back} based on the simulation data. We can calculate a value for d_n and the updated value for u_n using the following steps after computing u_n:

If $u_n > b \times d^{sta}$ Then

$d_n = d_{(n-1)} + u_n - b \times d^{back}$

$u_n = b \times d^{back}$

ElseIf $d_{(n-1)} > 0$ And $u_n < b \times d^{back}$ Then

(20) $d_n = d_{(n-1)} - (b \times d^{back} - u_n)$

$u_n = b \times d^{back}$

If $d_n < 0$ Then

$d_n = 0$

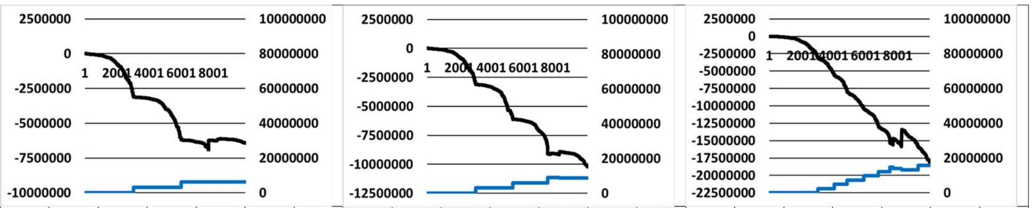
End If

Else

$d_n=d_{(n-1)}$

End If

Fig. 16 shows some examples. In this figure, the lower lines and the left axes of the graphs show d_n. Similar to Fig. 15, it was a promising idea, but it was not very successful. Unable to continue winning as expected, d_n increases again before it recovers.



(a)	*b*	-	A_{max}	96,200	c_b	718	(b)	*b*	-	A_{max}	96,200	c_b	595	(c)	*b*	-	A_{max}	97,100	c_b	595
R6	r_u	0.696	r_h	1.671	r_r	11.89	R6	r_u	0.84	r_h	1.176	r_r	3.331	R6	r_u	0.84	r_h	1.176	r_r	15.82
d^{sta}	2,200	d^{back}	5	data1			d^{sta}	2,200	d^{back}	50	data1			d^{sta}	2,200	d^{back}	500	data1		

Fig. 16. Examples of separating large uncollected amount.

Section 10 The multiple-lines model

In this section, we consider a multiline model. From the perspective of offering a variety of betting methods, horse racing is unique type of gamble. We can purchase various types of tickets besides the usual single-win ticket. Also, for a single-win ticket, we can bet on multiple horses at once. For those betting tickets where we can select multiple horses, such as with quinella and trifecta bets, there are also different ways to purchase those tickets, such as by box or wheel.

In this section, I focus on three strategies motivated by the goal of reducing the risk of doubling, as proposed in previous sections. In Section 10.1, I introduce a method for the purchase of multiple ranks. In Section 10.2, I introduce a method to divide bets among multiple lines rather than making consecutive bets. In Section 10.3, I introduce a hybrid method that combines these two methods. In each subsection, I introduce the strategy of using multiple lines when the uncollected amount grows large and applying the one-horse one-line model while that amount remains small. Finally, inspired by the cicada outbreak of 2024, I include Section 10.3.3 to discuss a method that uses a relatively large numbers of lines.

10.1 The one-line multiple-horses model

10.1.1 The simple multiple-horses model (Type 1L*m*H-A)

To begin with, let’s consider simple superposition.

We can formulate this using the steps outlined in Section 3.2:

Do STEP0 for every line.

$P^{\wedge}_n$:=0

For every line i

Do STEP1 to 6

STEP 6A Compute $P^{\wedge}_n := P^{\wedge}_n + P_n$

Next *i*

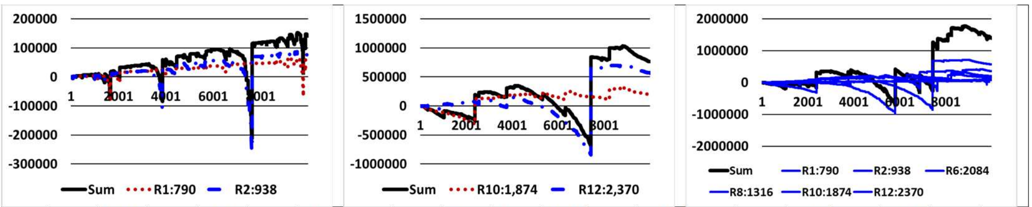
STEP7 Go to For-Loop (STEP1)

Table 36 shows a simple example for 3 lines. Each line contains a risk that the input amount becomes large. We can see that at race 13, R1 bets 900 yen, and R2 bets 400 yen. In reality, however, since the number of hits actually increases more than with the one-line model, it may feel as though the psychological stress has been reduced as a result. Yet, by continuing to bet, we may start to worry about the uncollected amounts for certain lines.

Table 36. Example of Type 1L*m*H-A

	b	c_b	1	2	3	4	5	6	7	8	9	10	11	12	13
Sum		39	-300	-210	-110	-60	90	140	1,100	1,460	1,820	1,680	2,050	1,980	580
R1	120	13	-100	-200	100	0	-100	-300	-700	-140	-340	-180	-480	-980	-1,880
R2	290	13	-100	190	90	340	690	1,040	940	840	740	540	340	40	-360
R3	510	13	-100	-200	-300	-400	-500	-600	860	760	1,420	1,320	2,190	2,920	2,820

Fig. 17 shows some examples of 1L*m*H-A. In this section, the betting count for each line is displayed in the legends for each graph. We use Type VI from Section 6. That is, we start the betting for all lines starting from the race whose odds are 1.5 or less, even if the rank of the line is 2 or greater. In other words, we bet on the same horse with 100×(line_num) yen.



(a)	A_{max}	48,800	$A^{\wedge}_{max}$	48,800	c_b	1,728	(b)	A_{max}	7,800	$A^{\wedge}_{max}$	8,500	c_b	4,244	(c)	A_{max}	48,800	$A^{\wedge}_{max}$	48,800	c_b	9,372
R1R2	r_u	11.63	r_h	31.94	r_r	109.1	R10R12	r_u	0.636	r_h	2.262	r_r	130.7	R 6 horses	r_u	0.224	r_h	10.13	r_r	121.9
Type VI	v	10					Type VI	v	10					Type VI	v	10				

Fig. 17. Examples of Type 1L*m*H-A.

10.1.2 Clear the uncollected amount when the total profit is updated (Type 1L*m*H-B)

In the multiple-lines model, to prevent generating a large uncollected amount, we can design a model which clears the entire uncollected amount when the total profit is updated. There are cases where the value of $A_{\max}$ in this model is actually larger than that for Type 1L*m*H-A. However, it is mentally relaxing to be able to clear the uncollected amount when we can do so.

We can formulate these steps by replacing STEP 6A of the previous subsection as follows:

STEP 6B Compute $P^{\wedge}_{n} := P^{\wedge}_{n} + P_n$

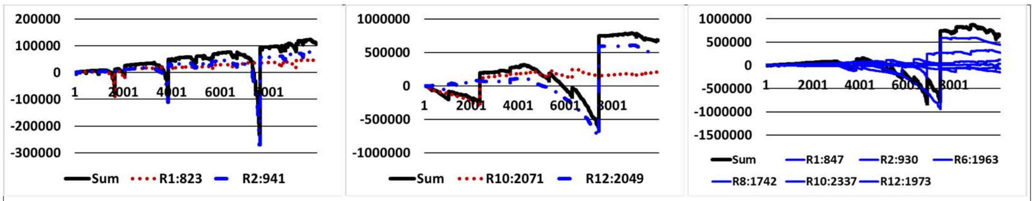
If $P^{\wedge}_{n} > P^{\wedge☆}$ then $P^{\wedge☆} := P^{\wedge}_{n}$, u_n=0 for all line *i*.

Table 37 shows an example using the same data as Table 36. From this table we can see that at race 13, R1 bets 300 yen and R2 bets 100 yen. These values are smaller than their respective values in Table 36. It seems that the risk has been reduced over a small number of races.

Table 37. Example of 1L*m*H-B

	b	c_b	1	2	3	4	5	6	7	8	9	10	11	12	13
Sum		39	-300	-210	-110	-60	90	240	770	650	1,110	950	1,620	2,150	1,850
R1	120	13	-100	-200	100	0	-100	-200	-300	-220	-320	-280	-380	-480	-580
R2	290	13	-100	190	90	340	690	1,040	940	840	740	640	540	440	340
R3	510	13	-100	-200	-300	-400	-500	-600	130	30	690	590	1,460	2,190	2,090

Fig. 18 shows some examples of the same data as Fig. 17. The differences of $A_{\max}$ are small. This shows that when a long losing streak occurs, the combined profits of the other lines cannot cover the loss.



(a)	A_{max}	48,800	$A^{\wedge}_{max}$	48,800	c_b	1,764	(b)	A_{max}	7,000	$A^{\wedge}_{max}$	7,700	c_b	4,120	(c)	A_{max}	40,000	$A^{\wedge}_{max}$	40,000	c_b	9,792
R1R2	r_u	12.64	r_h	34.07	r_r	108.7	R10R12	r_u	1.359	r_h	3.228	r_r	131.8	R 6 horses	r_u	1.143	r_h	11.35	r_r	114
Type VI	v	10					Type VI	v	10					Type VI	v	10				

Fig. 18. Examples of Type 1L*m*H-B.

10.1.3 Skip all races until the total profit is updated (1L*m*H-C)

Furthermore, to reduce the betting count, we can consider a method in which each line skips all races after clearing its own uncollected amount until the total (sum) profit is updated.

We can compute this method by replacing (8) as follows:

$$\text{If } (o_n \geqq (b+100) \div 100 \text{ and } u_{(n-1)} > 0)$$
$$\text{or } (o_n \leqq \xi \text{ and } u_{(n-1)} \leqq 0 \text{ and } \sigma_n = 0$$
$$\text{and } P^{\wedge}_{(n-1)} = P^{\wedge☆})$$

(21) $$\text{then } (a_n = \lceil u_{(n-1)} \div b \rceil \times a_{init} \text{ If } a_n = 0 \text{ then } a_n = a_{init})$$

Else $a_n=0$

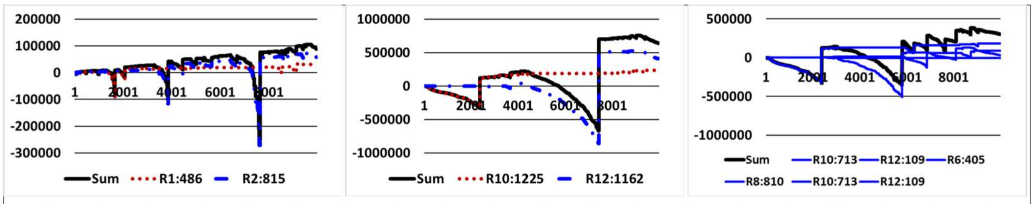
If $\sigma_{vn}=0$, bet on the most popular horse.

Table 38 shows an example using the same data as Tables 36 and 37. The race count c_b is reduced. I interpret this as a reduction of the risk of encountering a losing streak.

Table 38. Example of 1L*m*H-C

	b	c_b	1	2	3	4	5	6	7	8	9	10	11	12	13
Sum		28	-300	-210	-10	-110	-210	-310	1,150	1,030	1,590	1,430	2,200	2,730	2,430
R1	120	7	-100	-200	100	100	100	100	100	180	180	220	220	120	20
R2	290	8	-100	190	190	190	190	190	190	90	-10	-110	-210	-310	-410
R3	510	13	-100	-200	-300	-400	-500	-600	860	760	1,420	1,320	2,190	2,920	2,820

Fig. 19 shows some examples. Before this test, I expected that the value of A_{max} might be reduced. However, except for (c), differences of A_{max} in 10,000 races are either 0 or very small values. We cannot make up for the loss of a line's losing streak that easily.



(a)	A_{max}	48,800	$A^{\wedge}_{max}$	48,800	c_b	1,301	(b)	A_{max}	7,000	$A^{\wedge}_{max}$	7,000	c_b	2,387	(c)	A_{max}	23,200	$A^{\wedge}_{max}$	23,200	c_b	2,222
R1R2	r_u	16.6	r_h	30.98	r_r	107.7	R10R12	r_u	2.387	r_h	3.351	r_r	140.4	R 6 horses	r_u	1.485	r_h	6.345	r_r	118.2
Type VI'	v	10					Type VI'	v	10					Type VI'	v	10				

Fig. 19. Examples of Type 1L*m*H-C.

10.1.4 Bet on one horse while the uncollected amount is small (Type 1L1*m*H-B, 1L1*m*H-C)

In my opinion, the strategy of betting on one horse while the uncollected amount remains small is effective from the viewpoint of reducing the betting count. Similarly, I believe that using the multiple-horses model only when there is a large uncollected amount is also effective. When using the multiple-horses model, we can divide the uncollected amount between all the wagered horses. In this section, we divide by the rate of *b*.

Let c_ln be the coefficient for computing the line_no. Let max_div be the maximum number of divisions (maximum line_no). Finally, let $b^{\#}$ be the standard value of *b* for division. We can formulate these steps as follows:

   Line_num=1

  For *i* =1 to line_num

        Do STEP1 to 6

        STEP 6MB Compute $P^{\wedge}_n := P^{\wedge}_n + P_n$

If $P^{\wedge}_{n} > P^{\wedge☆}$ then $P^{\wedge☆} := P^{\wedge}_{n}$, u_n=0 for every line *i*.

Else <Division of u_n>

Next *i*

<Division of u_n>

For i=max_div to 1 Step -1

If line_num<i and sum_aft≧$b^{\#}$×(i-1)×c_ln then

Line_num=i

u_{ni}=sum_aft×${}_{i}b$÷$\Sigma_{j}\,{}_{j}b$

Exit For

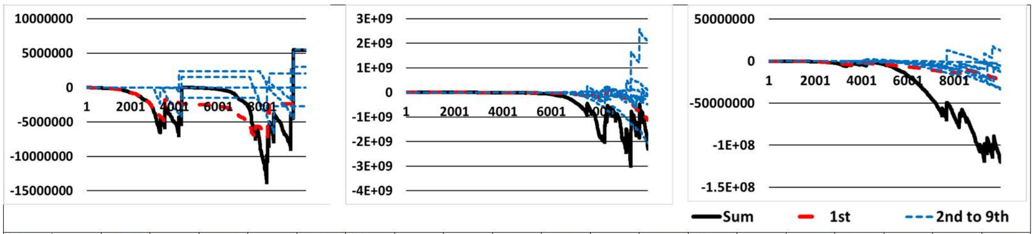
End If

Next i

Table 39 and Fig. 20 show some examples. The results are significantly different depending on the selection of ranks and their order. How to select these configurations is a topic for future study.

Table 39. Examples of 1L1*m*H-B and 1L1*m*H-C

	Type	1L1*m*H-B					1L1*m*H-C				
	c_ln	1,000	1,000	1,000	500	800	1,000	1,000	1,000	500	800
Order and c_b	1st	R6:785	R6:695	R6:695	R6:735	R6:695	R6:677	R6:695	R6:688	R6:723	R6:672
	2nd	R7:346	R5:532	R1:347	R7:466	R7:690	R7:191	R5:524	R1:308	R7:432	R7:585
	3rd	R8:169	R4:352	R2:420	R8:232	R8:388	R8:159	R4:349	R2:382	R8:227	R8:388
	4th	R9:199	R3:334	R3:353	R9:393	R9:638	R9:59	R3:334	R3:352	R9:344	R9:595
	5th	R10:92	R2:387	R4:360	R10:273	R10:406	R10:92	R2:371	R4:356	R10:269	R10:406
	6th	R11:0	R1:324	R5:529	R11:359	R11:467	R11:0	R1:303	R5:506	R11:355	R11:467
	7th	R12:0	R7:639	R7:659	R12:292	R12:380	R12:0	R7:631	R7:610	R12:292	R12:379
	9th	R13:0	R8:377	R8:386	R13:23	R13:32	R13:0	R8:377	R8:386	R13:23	R13:32
	9th	R14:0	R9:674	R9:701	R14:264	R14:358	R14:0	R9:669	R9:691	R14:242	R14:354
	Σ	Σ:1591	Σ:4314	Σ:4450	Σ:3037	Σ:4054	Σ:1178	Σ:4253	Σ:4279	Σ:2907	Σ:3878
	A_{max}	99,500	1.8E+7	1.5E+7	51,000	1.5E+5	99,500	1.8E+7	1.5E+7	51,000	1.6E+5
	$A^{\wedge}_{max}$	2.5E+5	9.2E+7	7.9E+7	3.6E+5	1.1E+6	2.5E+5	9.2E+7	7.9E+7	3.6E+5	1.1E+6
	r_u	0.628	0.139	0.134	0.428	0.148	0.848	0.141	0.14	0.447	0.154
	r_h	1.697	3.708	3.797	0.921	0.764	1.697	3.667	3.692	0.963	0.67
	r_r	113.347	84.691	84.699	58.752	56.229	113.401	84.691	84.699	58.758	56.188



(a)	A_{max}	99,500	$A^{\wedge}_{max}$	2.5E+5	c_b	1,178	(b)	A_{max}	1.8E+7	$A^{\wedge}_{max}$	9.2E+7	c_b	4,253	(c)	A_{max}	1.6E+5	$A^{\wedge}_{max}$	1.1E+6	c_b	3,878
6~14	r_u	0.848	r_h	1.697	r_r	113.4	6~1,7~9	r_u	0.141	r_h	3.667	r_r	84.69	6~14	r_u	0.154	r_h	0.67	r_r	56.19
data1	DivC=1,000						data1	DivC=1,000						data1	DivC=800					

Fig. 20. Examples of Type 1L1*m*H-C using data1.

10.2 The one-horse multiple-lines model

10.2.1 The simple multiple-lines model (1HmL-A)

The ideas from Sec. 10.1 can be applied to betting on a single horse. First, let's consider a simple model. That is, we bet on one rank horse by using multiple lines in a predetermined order. By this operation, if there are long losing streaks, we can reduce the length of the streaks to the overall length divided by the number of lines. Otherwise, there is the additional risk that each line may have a long losing streak. However, it is not known which line will perform best. Let's confirm by a simulation.

Let next_i be the line index of the next race. We can compute next_i as follows:

<Compute next_i>

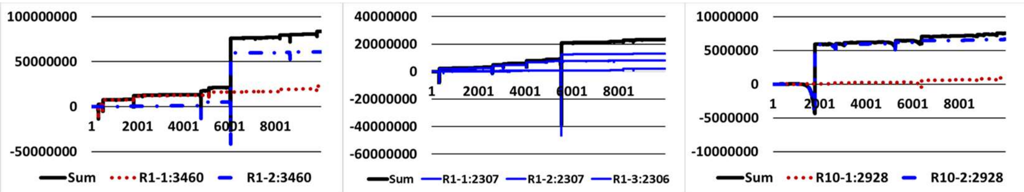
```
    next_i_n:= next_i_(n-1)+1
    if next_i_n>line_num then
        next_i_n:= 1
    end if
```

Table 40 is an example that shows three lines betting on the most popular horse. We bet on this one rank in the order of Line-1, Line-2, Line-3, Line-1, and so on. Since u_n and a_n are not displayed in this table, it may be a little difficult to interpret, but from race 11 to 13, R1 has a losing streak, and a_n for the one-line model reaches 300 yen. By contrast, the value of a_n remains at 200 yen for 1H3L-A.

Table 40. Example of profit change of 1H*m*L-A for R1 (*b*=120)

		c_b	1	2	3	4	5	6	7	8	9	10	11	12	13
1H1L-A		13	-100	-200	100	0	290	190	90	250	150	230	130	-70	-370
1H3L-A	Sum	13	-100	-200	-50	-150	140	40	-160	-80	-180	-20	-120	-320	-520
	Line-1	5	-100	-100	-100	-200	-200	-200	-400	-400	-400	-240	-240	-240	-440
	Line-2	4	0	-100	-100	-100	190	190	190	270	270	270	170	170	170
	Line-3	4	0	0	150	150	150	50	50	50	-50	-50	-50	-250	-250
	next_i		2	3	1	2	3	1	2	3	1	2	3	1	2

Fig. 21 shows some examples. Compared to Fig. 21(a) and Fig. 6(a), the value of $A_{\max}$ has increased. This means that one or more of the three lines experienced a long losing streak.



(a)	A_{max}	3.9E+7	$\hat{A}_{max}$	3.9E+7	c_b	6,920	(b)	A_{max}	3.9E+7	$\hat{A}_{max}$	3.9E+7	c_b	6,920	(c)	A_{max}	1.3E+5	$\hat{A}_{max}$	1.3E+5	c_b	5,856
R1m2	r_u	17.63	r_h	29.31	r_r	138.4	R1m3	r_u	13.53	r_h	29.31	r_r	115.8	R10m2	r_u	0.922	r_h	1.588	r_r	184.1
Type III							Type III							Type III						

Fig. 21. Examples of Type III of 1H*m*L-A.

10.2.2 Clear the uncollected amount when the total profit is updated (1H*m*L-B)

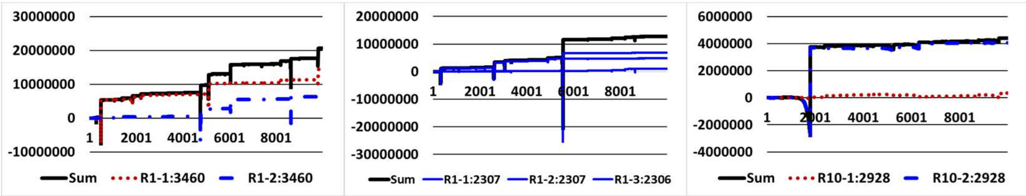
Similar to Section 10.1.2, we can design a model which clears the entire uncollected amount when the total profit is updated.

Table 41 shows an example. At the 5th race, the total profit is updated and the uncollected amounts of all lines are cleared. As a result, the betting amount decreases, and the loss of total profit at the 13th race is reduced compared to that in Table 40.

Table 41. Example of 1H*m*L-B for R1 (*b*=120)

	c_b	1	2	3	4	5	6	7	8	9	10	11	12	13
Sum	13	-100	-200	-50	-150	140	40	-60	20	-80	-40	-140	-340	-440
Line-1	5	-100	-100	-100	-200	-200	-200	-300	-300	-300	-260	-260	-260	-360
Line-2	4	0	-100	-100	-100	190	190	190	270	270	270	170	170	170
Line-3	4	0	0	150	150	150	50	50	50	-50	-50	-50	-250	-250
next_i		2	3	1	2	3	1	2	3	1	2	3	1	2

Fig. 22 shows some examples with the same conditions as Fig. 21. It seems that clearing the uncollected amount has resulted in a decrease in the value of $A_{\max}$.



(a)	$A_{\max}$	6.3E+6	$A^{\wedge}_{\max}$	6.3E+6	c_b	6,920	(b)	$A_{\max}$	2.1E+7	$A^{\wedge}_{\max}$	2.1E+7	c_b	6,920	(c)	$A_{\max}$	81,500	$A^{\wedge}_{\max}$	81,500	c_b	5,856
R1m2	r_u	17.05	r_h	29.31	r_r	126.9	R1m3	r_u	12.67	r_h	29.31	r_r	115.1	R10m2	r_u	1.007	r_h	1.588	r_r	192.6
Type III							Type III							Type III						

Fig. 22. Examples of Type III of 1H*m*L-B.

Tables 42 and 43 show the results for R1 to R16 using different numbers of multiple lines. Unfortunately, it is not possible to determine the optimal number of lines to use. This is due to the fact that there is still the possibility of a losing streak for each line, even after we reduce the risk of losing streaks through division of the one-line model.

Table 42. Results of 1H*m*L-B model for R1 to R7 (Type III, $b=R_{(50\%)}-100$)

Rank *m*	c_u	A_{max}	r_r	Rank *m*	c_u	A_{max}	r_r	Rank *m*	c_u	A_{max}	r_r
R1m1	1938	2.1E+7	123.507	R2m3	412	3.0E+7	108.54	R4m6	107	4.2E+5	159.641
R1m2	1180	6.3E+6	126.876	R2m4	332	1.2E+7	109.741	R4m7	106	1.7E+6	172.021
R1m3	877	2.1E+7	115.124	R2m5	269	476,000	117.025	R4m8	79	1.3E+6	145.097
R1m4	738	6.3E+6	115.579	R2m6	251	263,200	117.125	R4m9	73	4.5E+6	257.443
R1m5	585	6.3E+6	109.578	R2m7	209	3.0E+7	125.698	R4m10	77	1.2E+5	108.2
R1m6	518	3.4E+6	122.106	R2m8	187	6.8E+6	125.921	R5m1	319	2.8E+5	132.101
R1m7	477	8.1E+8	131.451	R2m9	164	353,900	131.298	R5m2	202	88,200	153.559
R1m8	418	3.4E+6	139.485	R2m10	144	1.3E+8	106.966	R5m3	120	4.5E+5	109.029
R1m9	393	4.4E+8	139.469	R2m11	139	3.8E+6	115.888	R5m4	126	1.7E+5	173.573
R1m10	339	6.3E+6	110.264	R2m12	129	860,800	114.239	R5m5	95	56,200	118.082
R1m11	325	1.0E+6	119.17	R2m13	81	3.8E+6	152.612	R5m6	68	42,900	115.497
R1m12	310	1.9E+6	116.259	R2m14	101	6.8E+6	112.436	R5m7	59	20,800	115.18
R1m13	281	6.3E+6	118.211	R2m15	89	5.4E+7	107.638	R5m8	59	1.3E+5	128.452
R1m14	261	1.9E+6	111.225	R2m16	92	2.1E+6	127.692	R5m9	59	1.4E+5	106.526
R1m15	229	3.4E+6	126.409	R2m17	84	5.1E+6	140.327	R5m10	55	56,200	119.038
R1m16	236	9.1E+9	163.618	R2m18	83	6.8E+6	101.977	R6m1	247	4.6E+5	120.525
R1m17	189	2.1E+7	159.089	R2m19	87	4.1E+7	137.687	R6m2	146	4.8E+7	132.888
R1m18	215	1.9E+6	114.648	R2m20	42	4.1E+7	155.132	R6m3	115	7.0E+5	126.05
R1m19	196	1.9E+6	110.934	R3m1	636	4.6E+7	124.738	R6m4	88	9.2E+5	140.425
R1m20	171	1.2E+7	134.485	R3m2	386	1.8E+6	146.065	R6m5	78	3.7E+5	124.501
R1m21	209	1.2E+7	104.586	R3m3	310	3.8E+7	111.676	R6m6	69	5.3E+5	107.181
R1m22	175	1.9E+6	179.407	R3m4	234	5.2E+5	163.448	R6m7	65	9.2E+5	100.516
R1m23	149	7.1E+7	151.512	R3m5	205	5.2E+5	135.349	R6m8	47	1.3E+5	118.712
R1m24	154	3.4E+6	126.229	R3m6	163	3.6E+5	126.459	R6m9	55	1.2E+5	145.293
R1m25	146	3.4E+6	129.808	R3m7	143	1.3E+6	125.579	R6m10	38	1.0E+7	108.902
R1m26	131	3.4E+6	133.735	R3m8	147	50,800	118.93	R7m1	186	6.0E+5	116.179
R1m27	165	559,000	118.779	R3m9	116	4.4E+5	121.832	R7m2	117	55,100	120.213
R1m28	149	3.4E+6	139.416	R3m10	94	3.1E+6	155.275	R7m3	75	3.5E+5	109.364
R1m29	115	2.4E+8	106.429	R3m11	107	1.0E+5	118.191	R7m4	64	4.8E+6	110.06
R1m30	122	2.1E+7	137.529	R3m12	105	72,600	112.706	R7m5	50	1.5E+6	147.159
R1m31	135	3.4E+6	134.197	R4m1	441	5.5E+7	109.343	R7m6	51	1.8E+5	127.914
R1m32	145	1.9E+6	121.056	R4m2	269	3.3E+5	131.021	R7m7	39	2.2E+7	140.924
R1m33	117	166,300	116.288	R4m3	210	2.9E+5	121.533	R7m8	39	21,900	111.736
R2m1	976	5.1E+6	120.638	R4m4	151	4.0E+6	172.306	R7m9	33	7.1E+5	124.135
R2m2	568	1.7E+7	119.859	R4m5	123	3.1E+6	125.987	R7m10	33	1.2E+5	110.466

Table 43. Results of 1H*m*L-B model for R8 to R16 (Type III, $b = R_{(50\%)} - 100$)

Rank *m*	c_u	A_{max}	r_r	Rank *m*	c_u	A_{max}	r_r	Rank *m*	c_u	A_{max}	r_r
R8m1	139	7.3E+6	128.841	R11m1	68	1.1E+9	171.381	R14m1	19	18,200	120.13
R8m2	82	7.7E+5	158.378	R11m2	42	6.9E+5	126.132	R14m2	13	10,000	131.61
R8m3	64	2.3E+5	166.642	R11m3	28	7.1E+5	109.851	R14m3	9	14,500	181.332
R8m4	54	1.2E+5	119.333	R11m4	21	20,900	116.797	R14m4	6	2,700	100.714
R8m5	45	1.9E+5	106.849	R11m5	17	2.1E+5	115.928	R14m5	4	26,800	127.999
R8m6	41	57,300	75.229	R11m6	13	1.0E+6	166.026	R14m6	4	3,400	80.625
R8m7	26	9.9E+6	120.742	R11m7	13	4.7E+5	109.303	R14m7	4	1,000	105.379
R8m8	35	1.7E+5	125.585	R11m8	11	51,600	78.5	R14m8	3	1,200	85.865
R8m9	30	5.1E+6	12.039	R11m9	10	29,000	105.386	R14m9	1	8,600	47.462
R8m10	29	2.2E+5	147.732	R11m10	8	50,400	51.473	R14m10	1	5,200	62.045
R9m1	104	2.0E+5	179.829	R12m1	48	1.8E+6	127.258	R15m1	6	1.5E+8	163.05
R9m2	63	2.0E+5	118.589	R12m2	29	6.9E+6	148.479	R15m2	2	8.1E+8	0.018
R9m3	47	89,700	191.651	R12m3	19	1.2E+5	143.966	R15m3	2	5.8E+6	156.96
R9m4	39	1.7E+5	155.808	R12m4	12	60,700	123.996	R15m4	1	6.3E+5	1.992
R9m5	29	1.7E+5	105.145	R12m5	16	2.9E+5	160.312	R15m5	1	98,200	7.946
R9m6	25	25,400	130.547	R12m6	13	6.1E+5	195.425	R15m6	1	28,300	33.047
R9m7	22	4.0E+5	144.348	R12m7	15	3,000	98.01	R15m7	1	11,600	15.303
R9m8	22	59,600	119.312	R12m8	12	4.2E+5	16.085	R15m8	1	6,000	17.416
R9m9	13	1.3E+6	141.522	R12m9	11	6,600	95.982	R15m9	1	3,600	22.446
R9m10	13	1.6E+5	156.374	R12m10	12	69,100	52.968	R15m10	1	2,300	13.731
R10m1	92	2.3E+5	227.1	R13m1	34	3.0E+5	219.858	R16m1	8	16,700	240.792
R10m2	59	81,500	192.555	R13m2	23	9.1E+6	173.733	R16m2	7	1,000	135.253
R10m3	40	1.5E+5	116.558	R13m3	20	80,700	231.054	R16m3	6	500	129.741
R10m4	38	31,600	128.53	R13m4	16	50,400	135.535	R16m4	4	400	106.513
R10m5	24	1.5E+5	325.955	R13m5	11	60,600	213.247	R16m5	4	200	98.347
R10m6	21	38,600	142.702	R13m6	16	7,300	115.193	R16m6	3	200	97.368
R10m7	23	4,900	123.482	R13m7	11	9,400	145.619	R16m7	3	200	77.868
R10m8	14	2.2E+5	113.773	R13m8	8	6,600	152.279	R16m8	3	200	80.976
R10m9	23	4,600	123.858	R13m9	12	3,300	109.702	R16m9	3	100	82.527
R10m10	16	2,900	111.306	R13m10	11	2,000	102.929	R16m10	3	100	82.527

10.2.3 Skip all races after a win until the total profit is updated (1H*m*L-C)

The strategy of skipping all races after the uncollected amount reaches zero is effective in order to obtain large profits. Alternatively, it is also effective for the remaining lines to continue betting until there is a win. By this operation, the risk of a losing streak for the remaining lines might be increased, however, we can expect to see a large profit.

We can compute next_i as follows:

<Compute next_i for 1H*m*L-C model>

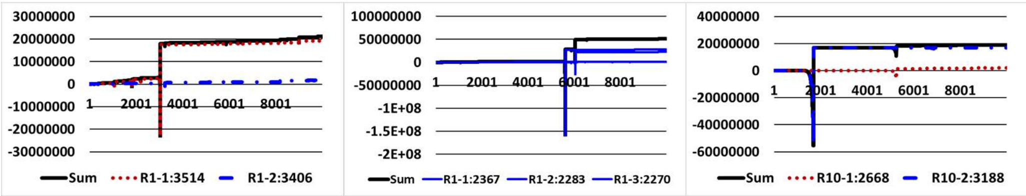
```
If a_{n next_i(n-1)}>0 then
   For i=Line_num to 1 Step -1
        If next_i(n-1)+i>line_num then
                If u_{n next_i(n-1)+i-line_num}>0 then
                        next_i_n:= next_i_(n-1)+i
                End if
        Else
                If u_n next_i_(n-1)+i>0 then
                        next_i_n:= next_i_(n-1)+i
                End if
        End if
   Next i
 Else
   next_i_n:= next_i_(n-1)
End if
```

Table 44 shows an example using the same data as Tables 40 and 41. We can see that line-1 encounters a losing streak, and the total profit at the 13th race drops rapidly. However, this amount becomes the source of funds for generating large returns.

Table 44. Example of 1H*m*L-C for R1 (b=120)

	c_b	1	2	3	4	5	6	7	8	9	10	11	12	13
Sum	13	-100	-200	-50	-150	140	40	-60	20	-80	-40	-240	-340	-740
Line-1	6	-100	-100	-100	-200	-200	-300	-300	-300	-400	-400	-600	-600	-1000
Line-2	5	0	-100	-100	-100	190	190	90	90	90	130	130	30	30
Line-3	2	0	0	150	150	150	150	150	230	230	230	230	230	230
next_i		2	3	1	2	1	2	3	1	2	1	1	1	1

Fig. 23 shows some examples using the same data as Fig. 21 and Fig. 22. Unfortunately, the results are worse.



(a)	A_{max}	2.1E+7	$A\hat{}_{max}$	2.1E+7	c_b	6,920	(b)	A_{max}	1.3E+8	$A\hat{}_{max}$	1.3E+8	c_b	6,920	(c)	A_{max}	1.6E+6	$A\hat{}_{max}$	1.6E+6	c_b	5,856
R1m2	r_u	21.07	r_h	29.31	r_r	127.6	R1m3	r_u	17.21	r_h	29.31	r_r	114.4	R10m2	r_u	1.195	r_h	1.588	r_r	128.8
Type III							Type III							Type III						

Fig. 23. Examples of Type III of Strategy 1H*m*L-C.

Tables 45 and 46 show the results for R1 to R16 using different numbers of multiple lines. Similar to Tables 42 and 43, it is not possible to determine the optimal numbers of lines that would be best to use, unfortunately.

Table 45. Results of 1H*m*L-C model for R1 to R7 (Type III, b=$R_{(50\%)}$-100)

Rank m	c_u	A_{max}	r_r	Rank m	c_u	A_{max}	r_r	Rank m	c_u	A_{max}	r_r
R1m1	1944	2.1E+7	123.517	R2m3	587	2.6E+7	134.434	R4m6	188	8.5E+6	164.563
R1m2	1458	2.1E+7	127.64	R2m4	505	6.8E+6	116.796	R4m7	174	4.5E+6	119.342
R1m3	1191	1.3E+8	114.429	R2m5	445	3.8E+6	124.647	R4m8	164	4.5E+6	142.654
R1m4	1011	3.9E+7	129.863	R2m6	408	2.2E+7	107.635	R4m9	147	1.3E+6	130.45
R1m5	911	1.2E+7	133.67	R2m7	361	5.8E+8	102.839	R4m10	140	1.1E+6	110.924
R1m6	811	3.5E+6	125.243	R2m8	324	5.1E+6	113.6	R5m1	319	2.8E+5	132.121
R1m7	745	6.3E+6	118.94	R2m9	312	6.8E+6	140.103	R5m2	243	2.6E+5	119.704
R1m8	683	3.3E+6	123.975	R2m10	273	1.6E+6	147.631	R5m3	193	2.4E+5	139.557
R1m9	647	3.9E+7	133.465	R2m11	266	4.2E+6	109.397	R5m4	165	4.1E+5	121.765
R1m10	604	1.9E+6	124.382	R2m12	261	1.4E+9	102.686	R5m5	139	3.4E+5	175.768
R1m11	562	7.2E+7	144.715	R2m13	237	1.7E+7	130.36	R5m6	132	9.2E+5	113.872
R1m12	522	6.3E+6	125.147	R2m14	227	1.7E+7	168.81	R5m7	120	1.4E+6	109.267
R1m13	502	1.9E+6	118.66	R2m15	219	3.8E+6	110.435	R5m8	104	2.7E+6	46.67
R1m14	470	1.9E+6	126.995	R2m16	204	3.5E+5	130.904	R5m9	92	1.2E+6	84.567
R1m15	446	1.9E+6	122.284	R2m17	189	2.1E+6	131.299	R5m10	83	2.2E+5	120.258
R1m16	441	3.9E+7	114.721	R2m18	200	1.4E+7	108.579	R6m1	247	4.6E+5	120.539
R1m17	410	1.9E+6	115.017	R2m19	187	1.2E+7	121.782	R6m2	183	3.0E+5	132.912
R1m18	415	2.4E+8	141.176	R2m20	180	3.0E+7	143.491	R6m3	147	1.3E+6	114.269
R1m19	397	1.0E+6	123.75	R3m1	636	4.6E+7	124.738	R6m4	129	6.0E+6	102.656
R1m20	387	6.3E+6	114.486	R3m2	461	3.8E+7	140.878	R6m5	116	2.6E+7	106.891
R1m21	365	7.2E+7	116.923	R3m3	376	1.6E+7	128.909	R6m6	104	4.5E+6	121.274
R1m22	366	6.3E+6	112.488	R3m4	334	1.8E+6	118.957	R6m7	97	1.8E+6	129.012
R1m23	327	1.3E+8	148.394	R3m5	294	7.6E+6	108.981	R6m8	88	3.0E+5	118.094
R1m24	328	1.0E+6	126.526	R3m6	262	8.9E+5	138.846	R6m9	85	2.8E+5	133.34
R1m25	319	3.9E+7	111.945	R3m7	243	5.3E+6	119.093	R6m10	75	9.0E+7	106.845
R1m26	317	1.9E+6	131.533	R3m8	229	3.0E+5	119.372	R7m1	186	6.0E+5	116.205
R1m27	279	6.3E+6	120.688	R3m9	219	6.2E+5	117.982	R7m2	135	1.4E+5	131.047
R1m28	280	1.2E+7	104.875	R3m10	203	6.2E+5	124.28	R7m3	114	3.7E+5	127.723
R1m29	272	6.3E+6	111.506	R3m11	189	3.1E+6	125.761	R7m4	90	94,900	148.332
R1m30	270	6.3E+6	124.412	R3m12	185	6.3E+6	117.927	R7m5	84	3.1E+6	135.779
R1m31	260	5.0E+9	149.945	R4m1	441	5.6E+7	109.344	R7m6	71	9.8E+5	119.182
R1m32	273	7.2E+7	146.186	R4m2	326	1.1E+6	145.239	R7m7	69	6.7E+5	119.729
R1m33	255	1.0E+6	137.316	R4m3	267	6.1E+5	124.97	R7m8	59	2.1E+6	123.427
R2m1	976	5.1E+6	120.638	R4m4	230	1.1E+6	146.273	R7m9	55	6.7E+5	122.023
R2m2	723	1.7E+7	120.153	R4m5	209	3.5E+6	175.68	R7m10	48	1.5E+5	120.012

Table 46. Results of 1H*m*L-C model for R8 to R16 (Type III, $b=R_{(50\%)}$-100)

Rank m	c_u	A_{max}	r_r	Rank m	c_u	A_{max}	r_r	Rank m	c_u	A_{max}	r_r
R8m1	139	7.3E+6	128.841	R11m1	68	1.1E+9	171.381	R14m1	19	18,200	120.127
R8m2	103	1.5E+7	126.043	R11m2	47	3.2E+9	171.427	R14m2	15	1.7E+5	118.723
R8m3	86	1.3E+5	147.672	R11m3	41	60,100	120.729	R14m3	11	61,100	106.039
R8m4	75	2.6E+5	124.492	R11m4	31	1.6E+6	152.877	R14m4	8	53,500	107.443
R8m5	66	1.5E+5	134.519	R11m5	26	4.8E+6	152.355	R14m5	7	1.2E+5	129.555
R8m6	57	8.4E+5	110.123	R11m6	22	1.4E+5	96.935	R14m6	8	1.4E+5	185.059
R8m7	50	5.4E+5	172.198	R11m7	22	1.1E+5	118.698	R14m7	8	10,100	118.044
R8m8	46	27,200	124.384	R11m8	18	20,400	108.111	R14m8	6	4,200	105.064
R8m9	45	2.5E+5	129.361	R11m9	20	2.7E+5	104.611	R14m9	5	2,200	104.189
R8m10	43	2.4E+5	156.27	R11m10	16	44,400	112.785	R14m10	4	1,200	102.431
R9m1	104	2.0E+5	179.829	R12m1	49	1.8E+6	127.249	R15m1	6	1.5E+8	163.05
R9m2	79	4.9E+5	184.815	R12m2	37	1.7E+7	100.846	R15m2	4	1.3E+6	39.433
R9m3	62	8.3E+5	149.517	R12m3	28	1.0E+7	188.433	R15m3	3	9.0E+8	163.061
R9m4	50	1.7E+5	122.673	R12m4	26	1.8E+5	144.295	R15m4	2	78,400	67.944
R9m5	42	1.3E+7	142.607	R12m5	23	3.1E+6	248.98	R15m5	2	1.1E+5	104.71
R9m6	39	1.7E+6	172.623	R12m6	19	52,600	121.537	R15m6	2	2.5E+10	163.053
R9m7	35	3.3E+5	147.878	R12m7	19	39,700	115.548	R15m7	1	5.5E+7	15.196
R9m8	33	2.4E+5	200.832	R12m8	18	1.4E+6	103.203	R15m8	1	9.2E+5	14.07
R9m9	31	4.4E+6	238.045	R12m9	17	3.3E+6	195.422	R15m9	1	88,200	23.314
R9m10	27	51,400	124.462	R12m10	15	1.5E+5	53.583	R15m10	1	29,500	15.677
R10m1	92	2.3E+5	227.098	R13m1	34	3.1E+5	219.853	R16m1	8	16,700	240.792
R10m2	70	1.6E+6	128.833	R13m2	29	2.3E+6	222.964	R16m2	7	1,000	134.282
R10m3	56	4.0E+5	145.595	R13m3	25	1.3E+6	102.722	R16m3	5	600	123.565
R10m4	47	8,900	123.852	R13m4	20	2.7E+6	280.114	R16m4	5	2,800	119.823
R10m5	44	10,300	121.069	R13m5	19	71,700	172.187	R16m5	4	200	104.505
R10m6	37	3,500	134.959	R13m6	17	27,500	179.533	R16m6	3	400	91.749
R10m7	34	4,900	124.128	R13m7	16	12,500	92.985	R16m7	3	300	87.201
R10m8	28	900	119.67	R13m8	15	3,800	103.5	R16m8	3	200	84.194
R10m9	24	2.7E+5	129.095	R13m9	15	2,000	94.057	R16m9	3	200	88.942
R10m10	29	1.1E+7	129.419	R13m10	14	3,800	113.962	R16m10	3	100	82.527

10.2.4 Bet on one horse while the uncollected amount is small (Type 1H1*m*L-B, 1H1*m*L-C)

Similar to Section 10.1.4, we can use the 1H*m*L model when the uncollected amount becomes large. As shown in the previous subsections, we cannot eliminate the risk of losing streaks. If that were possible, it would be better to continue using the one-line model until a long losing streak occurs.

Table 47 and Fig. 24 show some examples of the case where max_div=16 and c_ln=100,000. We can see that the value of *b* increases, while A_{max}, the number of lines, and c_b for each line decrease.

Table 47. Results of 1H1*m*L-B and 1H1*m*L-C for R6 with data1

Type		1H1*m*L-B						1H1*m*L-C					
b		1,390.0	1,739.3	2,088.6	2,437.9	2,787.2	3,136.5	1,390.0	1,739.3	2,088.6	2,437.9	2,787.2	3,136.5
c_b	Line1	3,749	2,885	1,157	1,124	382	335	3,855	2,898	1,160	599	379	335
	Line2	71	101	71	29	146	103	63	95	68	136	146	97
	Line3	47	74	2	0	125	61	38	67	2	89	126	61
	Line4	42	55	0	0	92	49	35	52	0	80	88	49
	Line5	30	40	0	0	54	40	31	40	0	50	61	40
	Line6	27	37	0	0	42	33	22	35	0	33	43	33
	Line7	25	31	0	0	37	27	19	33	0	29	41	27
	Line8	19	25	0	0	33	23	17	25	0	18	35	23
	Line9	17	23	0	0	14	13	12	23	0	15	10	13
	Line10	15	19	0	0	8	3	11	21	0	12	7	4
	Line11	12	14	0	0	5	0	9	17	0	7	4	0
	Line12	11	11	0	0	3	0	7	12	0	5	0	0
	Line13	10	8	0	0	0	0	6	9	0	3	0	0
	Line14	8	3	0	0	0	0	5	6	0	1	0	0
	Line15	6	2	0	0	0	0	4	2	0	0	0	0
	Line16	4	1	0	0	0	0	2	1	0	0	0	0
	Σ	4,093	3,329	1,230	1,153	941	687	4,136	3,336	1,230	1,077	940	682
A_{max}		2.3E+7	2.4E+6	99,300	97,500	1.0E+5	99,800	2.7E+7	2.7E+6	99,300	1.0E+5	1.0E+5	99,800
r_u		0.439775	0.180234	4.95935	5.290546	0.63762	0.873362	0.435203	0.179856	4.95935	1.207057	0.638298	0.879765
r_h		4.080137	3.214178	5.284553	5.464007	2.019129	1.310044	4.061896	3.207434	5.284553	2.321263	2.021277	1.319648
r_r		100.82	73.79	122.06	123.56	59.22	32.43	100.81	73.72	108.7	57.68	62.27	32.43

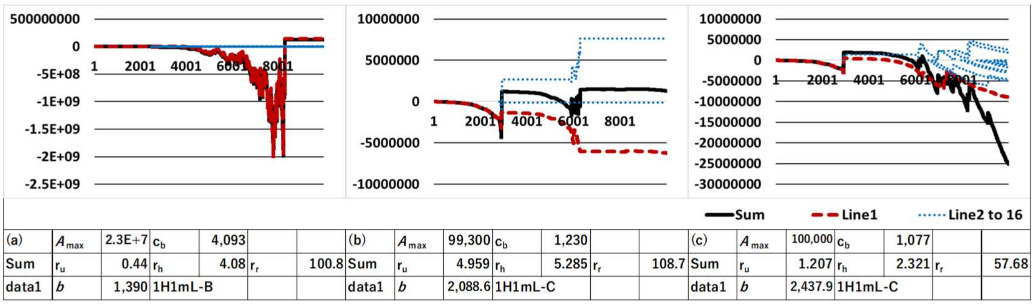


Fig. 24. Examples of 1H1*m*L-B and 1H1*m*L-C.

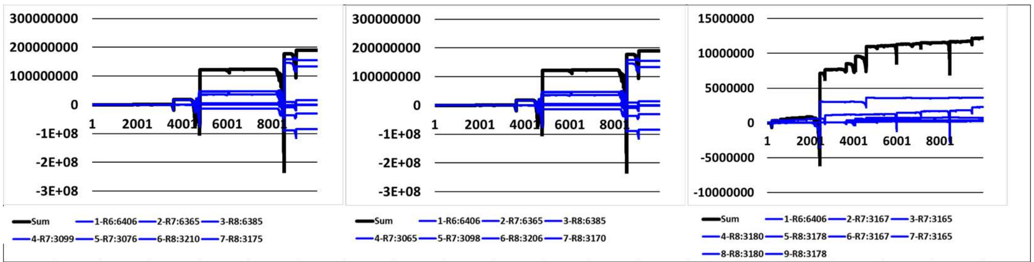
10.3 The multiple-lines multiple-sets model

10.3.1 The multiple-sets model (*m*L*m*S-B, *m*L*m*S-C)

We can combine the multiple-horses model and the multiple-lines model. As described in the previous subsections, we cannot completely eliminate the risk of generating a large uncollected amount through losing streaks. However, if we can design a multiple-lines multiple-sets model properly, then we might reduce the risk of a large uncollected amount by gaining larger profits from multiple sets.

We predetermine the number of sets. Furthermore, for each set, we determine the number of lines and the rank, and we apply the 1H*m*L model.

Fig. 25 shows some examples. These are not particularly good results. There are so many possible combinations that it is challenging to show a suitably representative example. Yet, I think it's an interesting idea and it might lead to positive results in the future.



(a)	A_{max}	3.2E+6	$A\^{}_{max}$	1.3E+7	c_b	31,716	(b)	A_{max}	3.2E+6	$A\^{}_{max}$	1.3E+7	c_b	31,695	(c)	A_{max}	3.2E+5	$A\^{}_{max}$	3.2E+5	c_b	31,786
	r_u	0.63	r_h	2.831	r_r	115.8	Type III	r_u	0.631	r_h	2.826	r_r	115.8	Type III	r_u	0.531	r_h	2.803	r_r	139.5
Type III	*m*L*m*S-B						Type III	*m*L*m*S-C						Type III	*m*L*m*S-C					

Fig. 25. Examples of Type III of *m*L*m*S-B and *m*L*m*S-C.

10.3.2 Use the multiple-sets model when the uncollected amount is large (1*m*L*m*S-B, 1*m*L*m*S-C)

The ideas from Sections 10.1.4 and 10.2.4 can be applied here. That is, an effective strategy is to bet on one horse with one line while u_n is small, and when u_n exceeds a predetermined value, we divide u_n into a predetermined number of lines.

We can formulate a replacement for <division of u_n> as follows:

<Division of u_n for 1H1*m*L-B and 1H1*m*L-C>

```
If max_aftb >= upper_lim And s_num(j) = max_s_num Then
    s_num(j) = max_s_num
    div_flg(j) = True
    line_num(j, max_s_num) = max_div(max_s_num)
    next_i(j - 1, max_s_num) = m_s(max_s_num)
Else 'If max_aftb >= upper_lim And s_num(j) = max_s_num Then

  For i2 = max_s_num To 2 Step -1
'       If max_aftb >= upper_lim And s_num(j) = i2 - 1 And s_num(j) < max_s_num Then
    If max_aftb >= upper_lim And s_num(j) = i2 - 1 And s_num(j) < max_s_num Then
      s_num(j) = i2
      div_flg(j) = True
      line_num(j, i2) = max_div(i2)
      next_i(j - 1, i2) = m_s(i2)
    End If
  Next i2
End If ' max_aftb >= upper_lim And s_num(j) = max_s_num Then
```

Furthermore, since the lines are bet in order by line-(Line_Num), such as line-1, line-2, and so on, we can formulate this rule by using a new variable next_i_n to represent the next line number of the line to bet for the n-th race. That is, we bet on the line-(next_$i_{(n-1)}$) for the n-th race, and we don't bet on any other lines for that race.

We can compute the value of next_i as follows:

<Compute next_i>

If line_num=1 then
  Next_i$_n$=1
Else
  If $a_{n\text{next\_i}(n-1)}$>0 then
    next_i$_n$:= next_i$_{(n-1)}$+1
    if next_i$_\text{n}$>line_num then
        next_i$_n$:= 1
    end if
  Else
    next_i$_n$:= next_i$_{(n-1)}$
  End if
End if

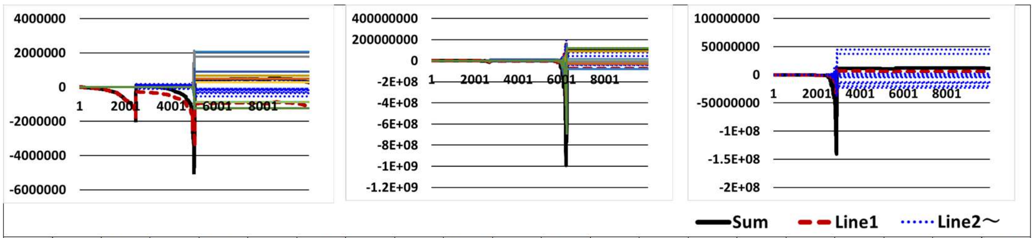
We can also apply the C model from Sec. 10.1.3. In this case, we compute the value of next_i as follows:
<Compute next_i for 1H1*m*L-C model>
If line_num=1 then
  Next_i$_n$=1
Else
  If $a_{n\text{next\_i}(n-1)}$>0 then
    For i=Line_num to 1 Step -1
      If next_i(n-1)+i>line_num then
        If $u_{n\text{next\_i}(n-1)\text{+i-line\_num}}$>0 then
          next_i$_n$:= next_i$_{(n-1)}$+i
        End if
      Else
        If $u_n$next_i$_{(n-1)\text{+i}}$>0 then
          next_i$_n$:= next_i$_{(n-1)}$+i

```
                End if
            End if
        Next i
    Else
        next_i_n := next_i_(n-1)
    End if
End if
```

Table 48 and Fig. 26 show some examples of 1*m*L*m*S-B and 1*m*L*m*S-C using data1 for this model. There are so many combinations that it is difficult to find suitably representative examples. However, if we can determine an appropriate rule, we may be able to make the value of A_{max} smaller.

Table 48. Examples of Type III of 1*m*L*m*S-B and 1*m*L*m*S-C (c_ln=999)

	(a)	(b)	(c)	(d)	€	(f)	(g)	(h)	(i)	(j)
model	1*m*L*m*S-B					1*m*L*m*S-C				
Sets	s1:R6:1 s2:R1:8 s3:R2:5 s4:R3:3 s5:R4:2 s6:R5:2 s7:R7:1 s8:R8:1	s1:R6:1 s2:R1:3 s3:R2:3 s4:R3:3 s5:R4:2 s6:R5:2 s7:R7:1 s8:R8:1	s1:R6:1 s2:R5:2 s3:R4:2 s4:R3:3 s5:R2:5 s6:R1:8 s7:R7:1 s8:R8:1	s1:R6:1 s2:R7:2 s3:R8:2 s4:R9:1 s5:R10:1 s6:R11:1 s7:R12:1 s8:R5:3	s1:R6:1 s2:R6:3 s3:R7:2 s4:R8:2 s5:R9:1 s6:R10:1 s7:R11:1 s8:R12:1	s1:R6:1 s2:R1:8 s3:R2:5 s4:R3:3 s5:R4:2 s6:R5:2 s7:R7:1 s8:R8:1	s1:R6:1 s2:R1:3 s3:R2:3 s4:R3:3 s5:R4:2 s6:R5:2 s7:R7:1 s8:R8:1	s1:R6:1 s2:R5:2 s3:R4:2 s4:R3:3 s5:R2:5 s6:R1:8 s7:R7:1 s8:R8:1	s1:R6:1 s2:R7:2 s3:R8:2 s4:R9:1 s5:R10:1 s6:R11:1 s7:R12:1 s8:R5:3	s1:R6:1 s2:R6:3 s3:R7:2 s4:R8:2 s5:R9:1 s6:R10:1 s7:R11:1 s8:R12:1
A_{max}	94,600	4.5E+5	5.6E+5	99,800	99,700	1.1E+5	99,500	1.3E+7	5.3E+5	6.7E+5
$A^{\wedge}_{max}$	5.1E+5	2.9E+6	4.0E+6	1.3E+5	2.5E+5	5.4E+5	3.1E+5	7.9E+7	3.6E+6	4.6E+6
c_b	1,688	2,506	3,245	1,518	1,803	2,416	1,575	4,144	3,143	2,979
r_r	101.171	100.043	100.532	118.143	103.545	102.845	104.965	102.045	104.001	103.49
Line1	R6_1346	R6_1442	R6_1506	R6_1384	R6_1443	R6_1492	R6_1344	R6_1685	R6_1558	R6_1532
Line2	R1_12	R1_63	R5_165	R7_58	R6_53	R1_37	R1_13	R5_223	R7_93	R6_114
Line3	R1_12	R1_63	R5_165	R7_58	R6_53	R1_35	R1_10	R5_226	R7_146	R6_88
Line4	R1_11	R1_62	R4_141	R8_9	R6_52	R1_34	R1_14	R4_13	R8_148	R6_115
Line5	R1_10	R2_53	R4_139	R8_9	R7_50	R1_34	R2_17	R4_14	R8_159	R7_47
Line6	R1_10	R2_53	R3_82	R9_0	R7_50	R1_37	R2_14	R3_40	R9_291	R7_56
Line7	R1_10	R2_52	R3_81	R10_0	R8_30	R1_34	R2_15	R3_45	R10_196	R8_127
Line8	R1_10	R3_52	R3_80	R11_0	R8_29	R1_33	R3_9	R3_44	R11_189	R8_127
Line9	R1_10	R3_51	R2_49	R12_0	R9_34	R1_35	R3_11	R2_99	R12_176	R9_238
Line10	R2_17	R3_50	R2_48	R5_0	R10_9	R2_49	R3_14	R2_99	R5_101	R10_189
Line11	R2_16	R4_71	R2_48	R5_0	R11_0	R2_49	R4_19	R2_99	R5_72	R11_179
Line12	R2_16	R4_70	R2_48	R5_0	R12_0	R2_43	R4_19	R2_100	R5_14	R12_167
Line13	R2_15	R5_72	R2_47			R2_49	R5_15	R2_95		
Line14	R2_15	R5_71	R1_32			R2_47	R5_8	R1_64		
Line15	R3_16	R7_143	R1_32			R3_18	R7_24	R1_65		
Line16	R3_16	R8_138	R1_31			R3_16	R8_29	R1_64		
Line17	R3_16		R1_31			R3_20		R1_64		
Line18	R4_21		R1_31			R4_15		R1_64		
Line19	R4_20		R1_30			R4_9		R1_64		
Line20	R5_17		R1_30			R5_33		R1_64		
Line21	R5_17		R1_30			R5_30		R1_64		
Line22	R7_34		R7_199			R7_147		R7_429		
Line23	R8_21		R8_200			R8_120		R8_420		



(a)	A_{max}	9.5E+4	$A\hat{}_{max}$	5.1E+5	c_b	1,688	(b)	A_{max}	1.3E+7	$A\hat{}_{max}$	7.9E+7	c_b	4,144	(c)	A_{max}	6.7E+5	$A\hat{}_{max}$	4.6E+6	c_b	2,979
Sum	r_u	5.213	r_h	7.997	r_r	101.2	Sum	r_u	1.592	r_h	9.459	r_r	102	Sum	r_u	2.584	r_h	3.86	r_r	103.5
Table 41(a)							Table 41(h)							Table 41(j)						

Fig. 26. Examples of Type III of 1*m*L*m*S-B and 1*m*L*m*S-C.

Table 49 shows the results of 100,000 races. These are extremely disappointing results. In many cases, the values for A_{max} are not small. Furthermore, the values for r_r are below 100%. These are frightening results that make these models unappealing for practical application.

Table 49. Results of 100,000 races

		Fig.20(a)		Fig.24(b)		Fig.26(a)	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		12,500	136.3	12,500	136.3	12,500	136.3
Test data	data1	99,200	80.5	98,400	×38.4	94,600	101.1
	data2	99,200	89.1	67,700	80.9	80,800	102.3
	data3	◎3,800	◎143.7	◎3,800	◎143.7	◎3,800	◎143.7
	data4	99,200	91.8	1.5E+5	61.0	2.0E+6	102.0
	data5	×99,900	41.3	99,700	56.5	2.2E+6	101.1
	data6	×99,900	78.6	81,600	96.8	×1.6E+7	101.2
	data7	43,900	99.1	43,900	99.1	43,900	×99.1
	data8	30,100	112.9	30,100	112.9	30,100	112.9
	data9	99,200	×14.5	×5.0E+5	59.0	1.3E+6	100.0
	data10	46,700	124.2	46,700	124.2	71,700	132.5
maxA_{max}	min_r_r	99,900	14.5	5.0E+5	38.4	1.6E+7	99.1
avgA_{max}	avg_r_r	72,110	87.57	1.1E+5	87.3	2.2E+6	109.6
I_Z			8.0		40.0		1,273.6

10.3.3 Prime-numbered cicadas may give some hints

I was planning to finish this paper by writing about multiple lines for Section 10.3.2. However, in 2024, a fortuitous event occurred when a large number of prime-numbered cicadas emerged in the United States. The mechanism of this phenomenon is explained by Yoshimura et al. (2009). As for the method of doubling, I don't think this is a clear hint that can be applied directly. However, it is noteworthy that cicadas with longer periodic cycles appeared, but not those with shorter periodic cycles. For the time being, I tried running simulations using a relatively large number of lines.

In this section, I set three values for b. Let LP=$R_{(50\%)}$-100 and RP= $R_{(97.5\%)}$-100. Furthermore, using b_idx we compute the value of b as follows:

(22) b=(5—b_idx)÷5×LP＋b_idx÷5×LP (b_idx=0,1,2,3,4,5)

Table 50 shows some examples where we use 2 sets including 12, 13, 17 and 18 lines (13-year and 17-year are the cycles of the prime-numbered periodic cicadas that emerged in the US in 2024). In some cases, we can see that the value of $A_{\max}$ exceeds 100,000. However, compared to Fig. 25, $A_{\max}$ has improved dramatically.

Fig. 27 shows some examples of cases for b_idx=5 with type *m*L*m*S-B. Compared to the previous figures, we can see some zigzag patterns, but we do not see any deep valleys. I have not considered this thoroughly in this paper, but it seems that due to the increased number of lines, the other lines offer small profits frequently, even if there is a losing streak. Although cicadas use the properties of prime numbers, we cannot find an efficient application of prime numbers in these figures and this table.

Table 50. Results of 2 sets

		Type	*m*L*m*S-A			*m*L*m*S-B			*m*L*m*S-C		
set1	set2	b_idx	3	4	5	3	4	5	3	4	5
R1_17	R1_13	A_{max}	4.7E+5	34,700	35,700	1.2E+5	2,400	900	43,300	3.8E+5	21,000
		$A\hat{}_{max}$	4.7E+5	34,900	35,800	1.2E+5	3,200	1,100	44,300	3.8E+5	21,700
		c_b	3,230	1,744	936	3,230	1,744	936	3,230	1,744	910
		r_r	84.752	113.106	115.613	105.762	101.257	103.078	111.518	67.772	110.527
R1_17	R1_12	A_{max}	4.7E+5	34,700	2,500	4.7E+5	7,800	1,900	60,900	62,900	5,600
		$A\hat{}_{max}$	4.7E+5	35,000	2,600	4.7E+5	8,100	2,000	61,600	63,400	6,000
		c_b	760	1,744	936	3,230	1,744	936	3,221	1,744	936
		r_r	111.282	108.578	118.071	102.556	102.981	101.767	117.057	116.565	107.173
R1_18	R1_12	A_{max}	4.7E+5	34,700	12,400	4.7E+5	7,800	1,900	60,900	62,900	5,600
		$A\hat{}_{max}$	4.7E+5	35,400	12,500	4.7E+5	8,200	2,400	61,400	63,300	6,000
		c_b	762	1,744	936	3,222	1,744	936	3,222	1,744	936
		r_r	113.509	105.186	108.638	103.225	102.135	102.317	117.645	116.641	107.861
R1_17	R2_13	A_{max}	5.1E+5	19,600	5,000	2.6E+5	2,400	700	85,700	96,800	16,100
		$A\hat{}_{max}$	5.1E+5	19,600	5,000	2.6E+5	2,400	700	85,700	96,800	16,100
		c_b	2,999	1,797	1,017	3,140	1,797	1,017	3,126	1,727	1,017
		r_r	95.174	119.244	114.231	97.789	100.496	94.191	107.268	113.182	112.442
R1_17	R2_12	A_{max}	4.7E+5	23,000	5,000	4.7E+5	8,800	1,400	85,700	27,000	11,700
		$A\hat{}_{max}$	4.7E+5	25,400	5,000	4.7E+5	13,300	1,400	85,700	27,000	11,700
		c_b	3,140	1,797	1,017	3,140	1,797	1,017	3,125	1,744	1,017
		r_r	71.053	90.381	101.763	47.224	84.572	95.247	106.797	114.553	105.031
R1_18	R2_12	A_{max}	60,900	25,700	12,400	60,900	10,500	2,500	30,800	84,800	11,700
		$A\hat{}_{max}$	60,900	25,700	12,400	60,900	10,500	2,500	30,800	84,800	11,700
		c_b	3,140	1,797	1,017	3,140	1,797	1,017	3,085	1,736	1,017
		r_r	119.861	88.614	97.227	98.843	78.555	96.775	115.830	116.18	111.488

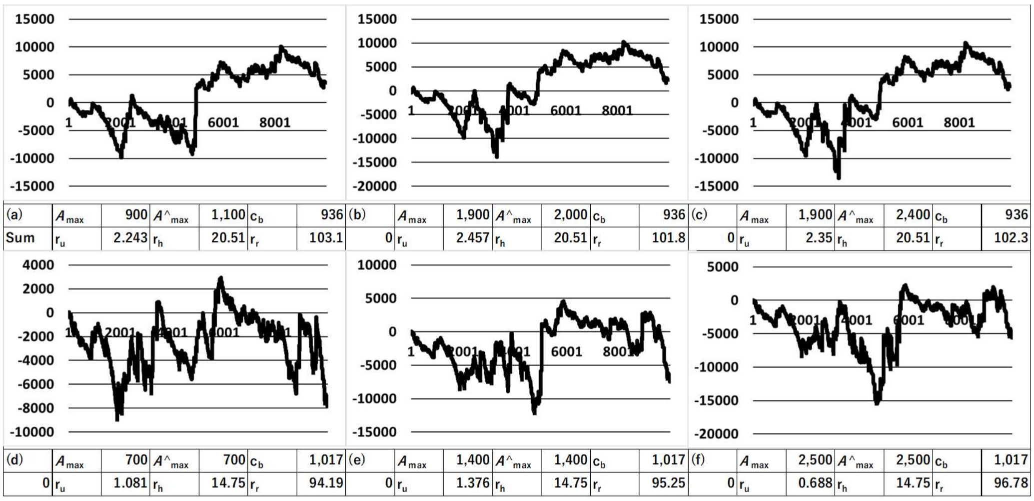
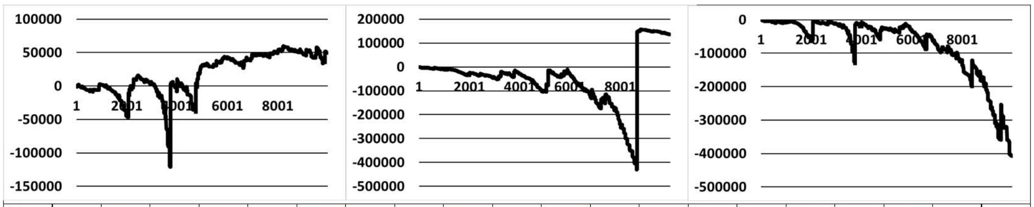


Fig. 27. Examples of Type *m*L*m*S-B, b_idx=5.

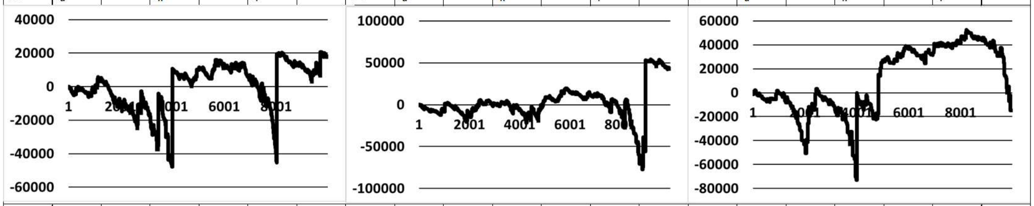
Tables 51 and 52, Fig. 28, and Fig. 29 show some examples using 6 sets. Through different choices of the rank and the number of lines, the results change significantly. There seems to be a correlation between the ranks and the number of lines: higher hit rate ranks tend to work well with more lines, while lower hit rate ranks are better suited to fewer lines. Furthermore, in an example such as Fig. 28(d), periodicity can be recognized. In this example, betting counts of set1 to set6 are 28, 43, 50, 67, 110 and 181 (or minus 1 of these values). We may be able to compute an appropriate number of lines and number of sets for each rank (or the betting strategy dependent on hit rates and recovery rates). These figures remind me of the superposition of periodic functions represented by Fourier series. I would like to delve into this more deeply as a topic for future study.

Table 51. Results of 6 sets of R1

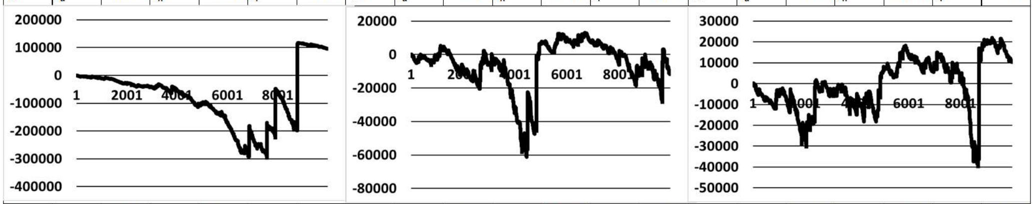
						Type	*m*L*m*S-A			*m*L*m*S-B			*m*L*m*S-C		
set1	set2	set3	set4	set5	set6	b_idx	4	5	6	4	5	6	4	5	6
R1_17	R1_13	R1_11	R1_7	R1_5	R1_3	A_{max}	4.7E+5	3.9E+5	78,900	1.2E+5	62,900	35,700	4.7E+5	62,900	27,400
						$A^{\wedge}_{max}$	5.1E+5	7.4E+5	82,600	1.2E+5	63,600	38,700	4.7E+5	64,400	28,400
						c_b	9,588	4,498	2,808	9,690	5,232	2,808	9,371	4,940	2,682
						r_r	99.260	89.36	113.532	109.370	104.802	107.598	107.623	109.926	110.54
R1_17	R2_13	R3_11	R4_7	R5_5	R6_3	A_{max}	5.1E+5	51,000	21,700	2.6E+5	31,200	21,700	3.4E+5	1.1E+5	32,200
						$A^{\wedge}_{max}$	5.1E+5	51,300	21,900	2.6E+5	31,200	21,900	3.4E+5	1.1E+5	32,200
						c_b	9,336	5,942	3,651	9,697	5,942	3,651	8,702	5,335	3,452
						r_r	84.999	99.786	109.414	109.644	112.258	111.256	104.583	113.977	104.317
R6_17	R5_13	R4_11	R3_7	R2_5	R1_3	A_{max}	5.7E+5	62,900	35,700	4.5E+5	46,700	35,700	1.7E+5	60,000	31,800
						$A^{\wedge}_{max}$	6.4E+5	62,900	35,700	4.5E+5	46,700	35,700	1.7E+5	60,000	31,900
						c_b	9,161	5,942	3,651	9,697	5,942	3,651	7,832	4,427	2,708
						r_r	78.295	108.919	72.763	102.592	101.63	72.763	112.348	108.297	116.416
R1_17	R2_13	R3_11	R1_7	R2_5	R3_3	A_{max}	1.4E+6	51,000	78,900	89,700	12,200	16,100	74,900	60,000	9,500
						$A^{\wedge}_{max}$	2.6E+6	51,400	79,100	89,800	13,700	16,300	75,200	60,200	9,600
						c_b	6,353	5,398	3,120	9,228	5,398	3,120	8,161	4,855	2,903
						r_r	84.025	97.551	120.036	108.704	105.393	103.278	109.939	114.601	107.924
R1_17	R2_13	R1_11	R2_7	R1_5	R2_3	A_{max}	5.4E+5	4.9E+5	18,000	5.4E+5	31,700	15,600	1.7E+5	84,800	46,500
						$A^{\wedge}_{max}$	9.8E+5	5.5E+5	18,400	5.6E+5	34,000	15,800	1.7E+5	85,200	46,800
						c_b	8,926	4,077	3,051	9,420	5,391	3,051	8,760	4,989	2,805
						r_r	94.044	84.054	115.669	107.160	106.023	106.933	106.565	106.661	113.572
R1_16	R1_12	R1_10	R1_8	R1_6	R1_4	A_{max}	4.7E+5	3.8E+5	16,100	4.7E+5	46,700	16,100	2.4E+5	1.1E+5	25,600
						$A^{\wedge}_{max}$	8.5E+5	3.8E+5	20,200	7.7E+5	56,100	20,200	2.4E+5	1.2E+5	28,500
						c_b	2,130	5,232	2,808	9,432	5,232	2,808	9,413	5,035	2,753
						r_r	111.847	110.048	110.257	102.582	105.585	97.399	112.513	111.794	112.417
R1_16	R2_12	R3_10	R4_8	R5_6	R6_4	A_{max}	5.7E+5	3.8E+5	8,300	6.0E+5	16,500	8,300	1.8E+5	62,900	86,700
						$A^{\wedge}_{max}$	2.1E+6	3.8E+5	10,400	9.4E+5	16,600	10,400	1.9E+5	62,900	86,700
						c_b	9,098	5,942	3,651	8,228	5,942	3,651	8,776	5,646	3,472
						r_r	78.770	69.798	104.4	62.833	80.453	107.874	109.234	106.597	105.532
R1_16	R2_12	R3_10	R1_8	R2_6	R3_4	A_{max}	5.4E+5	4.9E+5	9,500	1.7E+5	89,600	9,500	3.4E+5	1.1E+5	13,500
						$A^{\wedge}_{max}$	8.6E+5	9.5E+5	10,200	1.8E+5	90,700	10,200	3.4E+5	1.1E+5	13,700
						c_b	7,582	4,998	3,120	9,228	5,398	3,120	8,319	5,065	2,885
						r_r	84.798	87.993	94.672	103.892	134.419	97.844	113.200	117.126	107.245
R1_16	R2_12	R3_10	R1_8	R2_6	R3_4	A_{max}	1.0E+6	3.8E+5	6,600	21,200	25,700	6,600	2.4E+6	19,600	16,100
						$A^{\wedge}_{max}$	2.8E+6	3.8E+5	7,600	22,800	30,200	7,500	4.4E+6	20,000	16,300
						c_b	7,600	5,391	3,051	9,420	5,391	3,051	3,549	5,053	2,951
						r_r	79.636	108.698	110.936	106.193	103.693	102.006	81.514	109.486	110.267



(a)	A_{max}	35,700	$A^{\wedge}_{max}$	38,700	c_b	2,808	(b)	A_{max}	21,700	$A^{\wedge}_{max}$	21,900	c_b	3,651	(c)	A_{max}	35,700	$A^{\wedge}_{max}$	35,700	c_b	3,651
(g)	r_u	1.282	r_h	20.51	r_r	107.6	(h)	r_u	0.219	r_h	6.381	r_r	111.3	(i)	r_u	0.027	r_h	6.381	r_r	72.76



(d)	A_{max}	16,100	$A^{\wedge}_{max}$	16,300	c_b	3,120	(e)	A_{max}	15,600	$A^{\wedge}_{max}$	15,800	c_b	3,051	(f)	A_{max}	16,100	$A^{\wedge}_{max}$	20,200	c_b	2,808
(j)	r_u	0.544	r_h	11.47	r_r	103.3	(k)	r_u	0.753	r_h	14.75	r_r	106.9	(l)	r_u	1.068	r_h	20.51	r_r	97.4

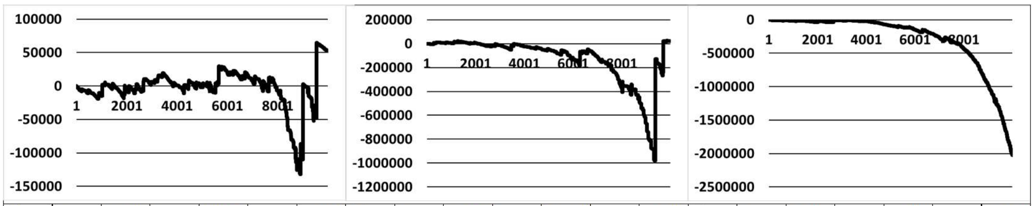


(d)	A_{max}	8,300	$A^{\wedge}_{max}$	10,400	c_b	3,651	(e)	A_{max}	9,500	$A^{\wedge}_{max}$	10,200	c_b	3,120	(f)	A_{max}	6,600	$A^{\wedge}_{max}$	7,500	c_b	3,051
(m)	r_u	0.109	r_h	6.381	r_r	107.9	(n)	r_u	0.512	r_h	11.47	r_r	97.84	(o)	r_u	0.753	r_h	14.75	r_r	102

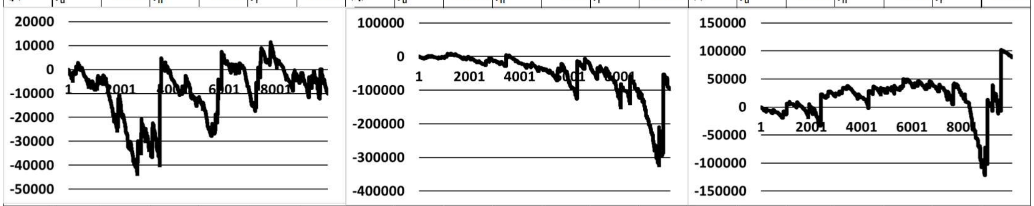
Fig. 28. Examples of Type *m*L*m*S-B, b_idx=6.

Table 52. Results of 6 sets of R2 and R3

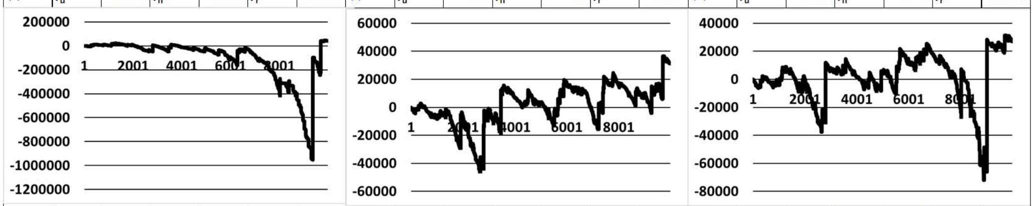
						Type	*m*L*m*S-A			*m*L*m*S-B			*m*L*m*S-C		
set1	set2	set3	set4	set5	set6	b_idx	4	5	6	4	5	6	4	5	6
R2_17	R2_13	R2_11	R2_7	R2_5	R2_3	A_{max}	5.5E+5	1.3E+5	18,000	5.4E+5	82,500	15,600	1.7E+5	60,000	42,400
						$A^{\wedge}_{max}$	6.8E+5	1.4E+5	21,800	5.6E+5	90,200	18,400	1.7E+5	61,900	43,200
						c_b	8,432	5,550	3,294	9,150	5,550	3,294	8,558	5,346	3,097
						r_r	98.515	94.307	110.019	105.533	69.298	107.123	103.714	112.811	106.525
R3_17	R3_13	R3_11	R3_7	R3_5	R3_3	A_{max}	5.5E+5	51,000	62,200	5.7E+5	19,900	62,200	50,900	21,900	26,800
						$A^{\wedge}_{max}$	8.5E+5	56,400	68,000	8.3E+5	28,800	65,900	52,300	22,800	28,200
						c_b	5,098	5,412	3,258	8,421	5,412	3,258	8,682	5,147	3,133
						r_r	98.181	79.644	91.796	101.174	110.498	101.115	124.987	111.825	106.089
R2_17	R3_13	R4_11	R5_7	R6_5	R7_3	A_{max}	74,900	38,500	24,300	38,400	12,300	24,300	2.9E+5	83,100	94,800
						$A^{\wedge}_{max}$	74,900	42,600	37,100	38,400	13,300	37,100	2.9E+5	84,500	94,800
						c_b	10,007	6,413	4,121	10,007	6,413	4,121	8,832	6,118	3,727
						r_r	116.519	75.604	24.973	108.812	63.384	24.973	118.322	110.631	111.47
R2_17	R3_13	R4_11	R2_7	R3_5	R4_3	A_{max}	5.7E+5	96,800	62,200	4.5E+5	16,800	2,100	4.1E+5	94,700	8,800
						$A^{\wedge}_{max}$	7.7E+5	96,900	62,500	4.5E+5	16,900	2,200	4.1E+5	94,900	8,900
						c_b	6,157	5,508	3,312	9,042	5,508	3,312	8,175	4,754	3,156
						r_r	82.696	82.329	105.342	117.679	102.606	97.743	106.785	134.401	109.402
R2_17	R3_13	R2_11	R3_7	R2_5	R3_3	A_{max}	6.6E+5	1.3E+5	34,500	3.9E+5	16,700	19,200	1.2E+5	60,000	8,300
						$A^{\wedge}_{max}$	1.9E+6	1.4E+5	35,100	3.9E+5	17,000	22,100	1.2E+5	60,400	8,600
						c_b	6,543	5,481	3,276	8,997	5,481	3,276	8,249	5,333	2,974
						r_r	82.575	95.786	82.637	143.891	99.235	92.946	109.412	117.249	111.791
R2_11	R2_9	R2_7	R2_5	R2_3	R2_2	A_{max}	5.4E+5	1.3E+5	18,000	5.4E+5	12,200	15,600	47,800	43,600	11,700
						$A^{\wedge}_{max}$	5.6E+5	1.4E+5	25,400	5.5E+5	17,400	18,700	55,100	45,100	12,500
						c_b	9,150	5,550	3,294	9,150	5,550	3,294	8,456	5,345	3,060
						r_r	115.006	96.519	115.873	103.851	106.498	110.617	106.647	110.16	110.073
R3_11	R3_9	R3_7	R3_5	R3_3	R3_2	A_{max}	9.2E+5	1.7E+5	62,200	9.3E+5	11,500	62,200	96,500	6,400	14,900
						$A^{\wedge}_{max}$	2.7E+6	2.1E+5	71,500	2.8E+6	18,200	69,500	1.0E+5	7,700	16,800
						c_b	6,257	5,412	3,258	6,257	5,412	3,258	8,524	5,275	3,001
						r_r	91.661	94.71	90.548	91.285	105.975	102.045	122.322	111.694	117.737
R2_11	R3_9	R4_7	R2_5	R3_3	R4_2	A_{max}	1.1E+6	2.3E+5	15,600	1.1E+6	2.3E+5	2,500	5.3E+5	4.8E+5	6,400
						$A^{\wedge}_{max}$	3.3E+6	2.3E+5	16,400	3.2E+6	2.3E+5	3,000	5.3E+5	6.7E+5	6,500
						c_b	6,269	5,508	3,312	6,269	5,508	3,312	2,926	4,041	2,965
						r_r	85.044	105.571	106.965	85.015	109.363	106.026	83.957	81.815	104.886
R2_11	R3_9	R2_7	R3_5	R2_3	R3_2	A_{max}	3.1E+6	1.7E+5	62,200	5.4E+5	43,600	13,500	5.4E+5	82,500	14,900
						$A^{\wedge}_{max}$	8.9E+6	1.7E+5	63,400	5.5E+5	48,000	13,700	6.7E+5	83,100	15,300
						c_b	6,432	5,481	3,276	8,997	5,481	3,276	3,325	5,011	3,064
						r_r	83.227	92.013	104.949	111.288	81.488	104.7	92.579	104.671	116.098
R3_11	R4_9	R3_7	R4_5	R3_3	R4_2	A_{max}	5.6E+5	2.3E+5	34,500	3.9E+5	2.3E+5	31,800	3.1E+5	88,800	6,400
						$A^{\wedge}_{max}$	7.5E+5	2.3E+5	35,000	3.9E+5	2.3E+5	32,300	3.1E+5	89,300	6,700
						c_b	4,532	5,487	3,321	8,988	5,487	3,321	7,518	5,043	2,971
						r_r	96.793	109.552	85.568	123.766	110.157	87.195	117.755	121.162	109.9



(a)	A_{max}	15,600	$A^{\wedge}_{max}$	18,400	c_b	3,294	(b)	A_{max}	62,200	$A^{\wedge}_{max}$	65,900	c_b	3,258	(c)	A_{max}	24,300	$A^{\wedge}_{max}$	37,100	c_b	4,121
(p)	r_u	0.333	r_h	9.836	r_r	107.1	(q)	r_u	0.214	r_h	5.34	r_r	101.1	(r)	r_u	0.024	r_h	3.518	r_r	24.97



(d)	A_{max}	2,100	$A^{\wedge}_{max}$	2,200	c_b	3,312	(e)	A_{max}	19,200	$A^{\wedge}_{max}$	22,100	c_b	3,276	(f)	A_{max}	15,600	$A^{\wedge}_{max}$	18,700	c_b	3,294
(s)	r_u	0.241	r_h	6.461	r_r	97.74	(t)	r_u	0.152	r_h	7.6	r_r	92.95	(u)	r_u	0.425	r_h	9.836	r_r	110.6



(d)	A_{max}	62,200	$A^{\wedge}_{max}$	69,500	c_b	3,258	(e)	A_{max}	2,500	$A^{\wedge}_{max}$	3,000	c_b	3,312	(f)	A_{max}	13,500	$A^{\wedge}_{max}$	13,700	c_b	3,276
(v)	r_u	0.214	r_h	5.34	r_r	102	(w)	r_u	0.332	r_h	6.461	r_r	106	(x)	r_u	0.396	r_h	7.6	r_r	104.7

Fig. 29. Examples of Type *m*L*m*S-B, b_idx=6.

Table 53 shows the results of 100,000 races using the parameters of data0. As with previous examples, the models that look promising may require more than 10,000 times the input amount.

Table 53. Results of 100,000 races

		R1_17 R1_13 A		R1_17 R1_13 B		R1_17 R1_13 C		R1_17 R1_13 R1_11 R1_7 R1_5 R1_3 A		R1_17 R1_13 R1_11 R1_7 R1_5 R1_3 B	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		35,700	115.613	900	103.1	21,000	110.5	78,900	113.5	35,700	107.6
Test data	data1	46,500	110.526	46,500	99.6	3.0E+5	116.3	3.6E+5	89.5	3.6E+5	88.1
	data2	21,000	71.703	9,500	73.0	7,300	103.4	46,500	100.0	12,400	99.6
	data3	1.0E+5	80.893	1.0E+5	80.5	16,100	106.0	1.0E+5	96.9	1.0E+5	96.7
	data4	9,500	102.36	4,300	91.6	16,100	115.6	75,900	113.9	46,500	108.2
	data5	9,500	78.043	900	95.8	35,700	113.2	78,900	99.2	12,400	104.5
	data6	7,300	102.502	2,500	98.2	4,300	110.9	2.3E+5	120.4	2.3E+5	117.5
	data7	21,000	84.37	21,000	100.0	4,300	104.4	27,400	101.4	5,600	103.0
	data8	27,400	105.686	16,100	105.1	60,600	110.3	3.0E+5	86.0	3.0E+5	76.5
	data9	7,300	106.473	1,900	97.9	4,300	115.1	3.0E+5	100.7	3.0E+5	102.4
	data10	12,400	109.902	12,400	104.8	9,500	105.3	60,600	113.9	60,600	110.2
maxAmax	min_rr	1.0E+5	71.703	1.0E+5	73.0	3.0E+5	103.4	3.6E+5	86.0	3.6E+5	76.5
avgAmax	avg_rr	26,470	95.2458	21,790	94.7	45,470	110.0	1.6E+5	102.2	1.4E+5	100.7
I_Z			288.0		11,422.2		1,411.9		458.3		1,013.4

		R1_17 R1_13 R1_11 R1_7 R1_5 R1_3 C		R1_17 R2_13 R3_11 R4_7 R5_5 R6_3 A		R1_17 R2_13 R3_11 R4_7 R5_5 R6_3 B		R1_17 R2_13 R3_11 R4_7 R5_5 R6_3 C	
		A_{max}	r_r	A_{max}	r_r	A_{max}	r_r	A_{max}	r_r
data0		27,400	110.5	21,700	109.4	21,700	111.3	32,200	104.3
Test data	data1	60,600	110.7	47,800	46.4	47,800	46.4	3.7E+5	34.0
	data2	2.3E+5	118.9	5,800	115.5	5,800	106.4	3.0E+5	105.8
	data3	1.3E+5	108.7	1.0E+5	91.8	12,400	107.9	60,600	105.5
	data4	5,600	114.4	22,000	105.8	22,000	108.5	60,600	111.5
	data5	9,500	107.2	61,900	143.6	60,000	144.2	98,300	113.0
	data6	3.4E+5	80.9	7,100	113.7	7,100	113.5	13,400	121.3
	data7	12,400	106.3	43,800	55.2	43,800	55.2	35,700	102.8
	data8	1.0E+5	103.4	27,400	109.9	6,300	105.1	2.7E+5	114.7
	data9	35,700	107.6	24,900	118.5	24,900	116.5	4.4E+5	72.7
	data10	3.0E+5	108.9	13,500	98.9	2,500	103.9	65,100	107.7
maxAmax	min_rr	3.4E+5	80.9	1.0E+5	46.4	60,000	46.4	4.4E+5	34.0
avgAmax	avg_rr	1.2E+5	106.7	35,700	99.9	23,260	100.8	1.7E+5	98.9
I_Z			1,252.6		473.7		276.5		1,365.5

11 End Talk

11.1 Conclusion

In this paper, I proposed a relaxation method for the rule of doubles in order to make a profit with low risk at a virtual racecourse. In these models, rather than using predictions, we continued to bet on the same rank or the same horse number. I designed several models by using the data gathered from 10,000 races. I confirmed the amount of risk present in the data through 100,000 races, with the forward-looking goal of practical use of the models in future races. We observed that, while a model may appear to be safe over a short timeframe, up to 10,000 times the input amount may be needed.

In Section 2, I explained the method of constructing the virtual horse racing data. A large amount of racing data is necessary for discussions regarding betting strategy. Although the accuracy may not be high enough, I believe it is a meaningful example.

In Section 3, I defined parameters and variables. I also introduced the standard model. The input amount is calculated by the unclaimed winnings and a predetermined coefficient. By this configuration, the rate of increase of the input amount can be slowed down.

In Section 4, I introduced good periods and bad periods in the standard model. I also explained the importance of setting coefficient b.

In Section 5, I proposed the reduction of betting frequency. By this operation, the maximum input amount, A_{max}, has improved from being an astronomically high value to a more realistic value.

In Section 6, I discussed increasing coefficient b when the uncollected amount becomes large.

Additionally, in Section 7, I proposed the strategy of lowering the horse rank as a workaround when the uncollected amount becomes even larger.

In Section 8, looking towards practical application, I showed the results of 100,000 races by applying a model that was found to be good in 10,000 races with initial parameters. This resulted in some cases where the input amount grows to 10,000 times its initial value. This is the most important conclusion of this paper.

Sections 9 and 10 were included as subjects for future work. In Section 9, I explained that we can not recover from a large loss by simple methods.

In Section 10, I introduced the one-line multiple-horses model, the one-horse multiple-lines model, and the multiple-lines multiple-sets model. These models seem attractive because (1) the multiple-lines model allows for dividing the uncollected amount, (2) the multiple-horses model allows for raising the hit rate, and (3) the one-horse multiple-lines model allows for slowing down the rate of increase of the uncollected amount. The multiple-lines model may be more effective if used when the uncollected amount becomes large. Further, when we use a large number of lines, the betting counts for each line are reduced, so we might be able to prevent sudden declines as seen in the previous sections. We may be able to develop a periodic system similar to those seen with massive outbreaks of periodic cicadas.

In consideration of actual practical application, there are many problems. First of all, I used artificial racing data. I don’t know how much of a difference there is compared to actual data. I gave examples with the assumption that I knew which methods have a recovery rate of 80%. But I don’t know how to bet in order to obtain an 80% recovery rate. Even if I found some data where the recovery rate is 80% within a certain time period, the next time period might not even reach 50%. In Japanese newspapers, tipsters use marks such as ◎, ○, ▲, and △, but these marks are often intended for use in key horse betting methods such as wheel and box. Some tipsters’ use of the mark ◎ might not even fulfill an 80% recovery rate. With wheel and box betting, the amount of the bet made at one time tends to be high, so these are not suitable for doubling if we do not have enough funds to cover the bet.

There is also a problem with the amount of data. I used data from 100,000 races for verifying the safety of the models. However, in some cases, many races were skipped. For example, with the N*n* strategy of Type III in Section 5.2, 88% of the races were skipped, as shown in Fig. 6(c). I would like to claim that I have generated a sufficient amount of test data. However, if someone pointed out that I only used 12% in some cases, I would have to acknowledge that. In addition, I don't know how many tests would be sufficient. Skipping races to reduce betting counts is the most important strategy in Section 5. This allows us to steer clear of the most easily avoidable losing streaks.

It's not just a financial issue, but also a psychological burden. As for myself, I put these methods into practice before writing this paper, and I felt a lot of mental stress. For the popular horses, we must set parameter b to a low enough value to obtain winnings. However, even if the hit rate is relatively high, the recovery rate might change. Even if the recovery rate is over 80% within a certain time period, it may be different in the next time period. I tried methods based on using the most popular horses and the prediction symbols in newspapers, but I often felt discouraged by sudden increases in the betting amount. Sufficient research and preparation is necessary. For the less popular horses, we can set parameter b to a large value, but we must wait a long time for a dark horse long shot to win. I once tried waiting through 48 races over two weekends for the third horse from the bottom (the third lowest-ranked horse) to win. I know that statistically it should occur at a 1% or 2% hit rate, but I don't know when it will occur. While the payoff is large, the psychological stress is unbearable and there is no estimate for how much time it will take. When I bet on the third horse from the bottom over a long period, it's not a problem if a popular horse comes in. That loss is endurable. But if the fourth horse from the bottom comes in, then I feel regret. If the second horse from the bottom comes in, it's more painful, and I regret choosing the third horse instead of the second horse. Repeat that experience over and over. If I had endured the trial and my horse eventually came in, I would have earned a big win. This was the case for just 48 races. I doubt whether waiting over a longer period is possible.

11.2 Future study

In Section 5, I focused on the reduction of the number of bets. I believe that this is one of the more effective methods. In televised programs, some tipsters will skip a number of races, and we can observe voters skipping some races as well. They must know from experience that skipping is an effective strategy. However, finding more effective methods to reduce the number of bets is a topic for future study.

In Section 10, I introduced the multiple-lines model. I believe that the one-line model is better than the multiple-lines model. However, the multiple-lines model is effective when the uncollected amount is large. Going forward, we might be able to create a periodic system.

In Section 5.4, I used constant betting data and proposed starting from low-odds races. Even if using the doubling method is theoretically safe, the psychological stress is inevitable, so we would like to avoid using doubling as much as possible. Ideally, we want to develop a winning strategy which does not require using the doubling method. We should only use doubling as a last resort when we cannot obtain winnings by constant betting. Developing a more effective combination method is a topic for future study.

In Section 9, I explained that recovering from a large loss is difficult. In the first place, we don't know the amount of funds we would need to obtain winnings. If there is a debt, we can pay it off in installments. However, in doubling, there is a risk that the divided uncollected amount may become large again. For now, it is important to develop methods such that uncollected amount remains under a predetermined value.

I believe that the methods proposed in this paper have properties similar to the iterative methods for computing $f(x)=0$ as with the Newton method. Hundreds of papers have been published in the field of numerical calculation. Studies of convergence conditions might contribute some improvement to the doubling method. Actually, while I was changing the value of b in Section 6, I was recalling the change of the slope of the Newton method.

In the field of stochastic processes, similar discussions often take place where convergence conditions are deeply discussed. As a result, I hope that we can find conditions in the future that allow us to obtain winnings from lower betting amounts. The rule of doubles was referred to as the Martingale method somewhat abruptly in the field of horse racing. However, this is a term that was originally used in the context of conditional probability. (See Nishiyama, 2021).

The figures for profit changes remind me of Fourier analysis, especially those of the multiple-lines model. We can interpret hit rates periodically and the multiple-lines model as a superposition of their functions.

Study of the Traveling Salesman Problem and Steiner's Problem might also contribute to future study. Heuristic algorithms are developed and productivity is largely improved. These ideas might also contribute to the doubling method.

In this paper, I constructed the data such that the average of returns is made close to actual Japanese races. However, I still think there is a big difference between these data and those of real races. The results based on these data should be taken with a grain of salt. Higher quality data will be needed for more accurate analysis.

Further, I only bet on single-win ticket types—that is, win bets. Different methods will be needed for ticket types such as place, quinella place, bracket quinella, quinella, exacta, trio, and trifecta. For other types of public gambling, such as boat racing and bicycle races, I think these methods can be applied by using the same parameters. However, from the viewpoint of acquiring historical race data, single-win tickets in horse racing are the easiest.

These methods may also be applicable to investments such as stocks, exchange rates, and futures trading. I don’t yet know how to set the parameters, though there might be some hints in the large amount of investment information that is updated daily.

Since I show examples generated by using artificial data, valid parameter values might differ. I avoided using real-world data because I didn’t know how effective the historical data was. I cannot bear any responsibility if impatient readers go bankrupt by applying a proposed model. I only believe that there must exist some methods and parameters that can increase winnings in horse racing. There are many factors at play. Basically, the strong horses win. Depending on the course, there are advantages and disadvantages for each horse number and each gate number. We can refer to statistical data for historical recovery rates and hit rates, and we can also consider the distance between the gate and the first corner. There is also a characteristic similar to the property of memorylessness. That is, the probability of extending the losing streak of the most popular horse is independent of the current length of its losing streak. As these properties are further and more deeply analyzed, the accuracy of forecasting for future investment should be improved.

11.3 Considering the emergence of winning methods

It is interesting to consider the world after the development of successful methods of winning.

I don’t wish for young people to quit their jobs and immerse themselves in horse racing. However, it would be nice to make life easier. Gambling has led to a lot of social problems including terrible tragedies, addiction, and dependence. I hope that successful methods of winning can help to prevent these unfortunate situations.

I think that when people stop excessively saving and start spending their money, it leads to enriching our country. First of all, 10% of sales from the purchase of horse racing tickets is paid to the national treasury in Japan. Not to mention, workers in the horse racing industry can be saved. Furthermore, in recent times, horses and horse racing have become factors in international trade. Stallions are traded actively, and many Japanese horses participate in overseas races. I hope that a Japanese horse will win the Prix de l'Arc de Triomphe title someday. I expect that the income from horse racing may also be able to provide support for the independence of poor countries.

On the contrary, will the value of the currency of countries who have horse racing decline against those of countries who do not? Or, will the value of money disappear? Will the value of food ever exceed the value of money? In ancient times, humans didn't use money. Animals other than humans also don't use money. When currency was developed, people thought it was convenient. We have inherited the tools of our predecessors. Meanwhile, the disadvantages of money have been neglected. Since food spoils, it is better to distribute it rather than having to store large quantities. Money doesn't rot, so it can be kept in banks for future use instead of being distributed to the poor. Can we call this convenient? Osamu Dazai claimed that if people didn't know that Mt. Fuji was famous, they wouldn't be impressed. Since everyone says it is beautiful, people are impressed by the mountain. (Having said that, I do love this mountain.) Similarly, just as our parents used money for convenience, we also use it without question. If aliens were observing this situation, what would they say? Why do they use money? Is money convenient? Is this despite the fact that many people are killed not by food or by love, but by money?

11.4 What horse racing teaches us

First of all, horse racing teaches us many variations of how to use money. I can enjoy a stable life with no debt by renting an inexpensive apartment. Not knowing a thing about horse racing, I could not take out a housing loan due to an indistinct feeling of anxiety. What would I do if something happened to me? Without overcoming this anxiety, I still lived in a lower-class apartment. This way of thinking changed after experiencing the ups and downs of income and expenses as a result of purchasing horse racing tickets. If I knew about horse racing and how to bet on horses to make money, I could take out a housing loan and acquire my dream home. Buying a house is a high-risk undertaking. However, I have learned how to compete. Because I'm no longer young, I cannot take out a loan. From this introspection, I have determined that if these methods become well-known, I would buy a home. This is my gamble. If I bought a horse, I could be the owner of a Prix de l'Arc de Triomphe prize horse. Even before winning a single bet, the gambler's fantasy is endless.

Further, horse racing tells us that our forecasting is pretty sloppy even after many attempts. It is said that the most popular horse loses over 60% of the time, and the recovery rate is about 80%. Although I don't intend to reject democracy, I think that the rights of election winners are just too expansive. Election winners simply gained popularity after completing training and paddock work. It is easy to know how many times the most popular horse wins by the fanfare of the trumpets. Yet how can we tell how many election winners end up leaving good results? Not to mention, voters must research candidates more seriously than they do horse races. Many gamblers check the historical race data for horse racing. Similarly, we need to reflect on our voting behavior and the performance of our candidates. Candidates who lose elections did not lose the race just because they did not receive enough votes. It would be beneficial to accumulate their accomplishments for the next election.

In horse racing, it is possible to recognize when profits are diminishing. We can adjust our betting methods, change our strategy, or eventually stop betting, as some do. If we seriously want to obtain profits, we can employ the PDCA (Plan-Do-Check-Act) cycle as we plot how the profit changes race-by-race. It is possible to discover some effective strategies in that cycle, such as the reduction of the number of betting tickets and betting amounts. On the other hand, as an example from daily life, even if we keep a household account ledger, it may not be possible to improve profits because there are too many factors involved. We may not realize that we are being held back by things that aren't actually important. Personally, I think this way of thinking has enabled me to throw away things that were not necessary to hold on to, and this improved my personal finances. Cardano said that not betting is the best strategy for gamblers. Similarly, not buying and not owning may be the best strategy for life.

Please forgive me for continuing to ramble on. I would also like to mention the use of historical data. In Section 8, I showed that the doubling models that show good results in data0 often lead to catastrophic results in 100,000 races. Can this be considered a special circumstance unique to doubling? We frequently discuss safety by using historical data. However, in the fields of climate and earthquakes, for example, we often experience damage that exceeds the limits of historical data. This suggests that it may be a mistake to argue for safety based solely on historical data. We must consider alternative approaches to demonstrating safety. In the case of horse racing, this is not a problem, because the only person who is harmed is the gambler themselves.

There are many people who don't participate in horse racing because they think they will lose money. I think it's true that you'll lose money. But if you try it and think that you're throwing away money, you'll discover something. I recommend that you decide on a time limit and a maximum amount and try it out.

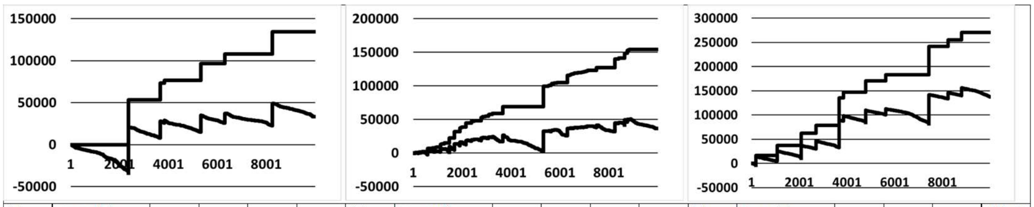
11.5 Background of this paper

Despite my poor English skills, there are three reasons why I wrote this paper in English.

The first reason is that I want people all over the world to know about these methods.

The second reason is to prevent the bankruptcy of impatient readers who come away with misunderstandings. In English, it would be difficult to extract an excerpt and implement it for Japanese people. On the other hand, for non-Japanese people, it is difficult to implement the proposed models because of the different currencies and circumstances around horse racing.

The third reason is tax. From the viewpoint of investment, taxes cannot be ignored. In Japan, the purchase cost of a losing betting ticket usually cannot be treated as an expense. According to the guidelines of the National Tax Agency, the conditions that allow for losing betting tickets to be treated as expenses are extremely strict. As long as this system is in place, if the profit from winning betting tickets exceeds 500,000 yen in one year (excluding losing betting tickets), then we must pay taxes calculated on this amount combined with other income. Fig. 30 shows some examples of cases where there are 10,000 races in a year. The upper lines are the profit changes for computing taxes, and the lower lines are the actual profits including the cost of losing betting tickets. Even if the lower lines fall below zero, if the upper lines surpass 500,000, then we must pay additional taxes. Since total taxes are dependent on all other income, they cannot be shown easily in figures. However, as shown in Section 8, I can claim that after one year, profits based on doubling might be negative, as we might have purchased more betting tickets that lost. Under these conditions, models based on doubling cannot be used for investment.



(a)	Fig. 11(c)				P^{tax}_{10000}	134,270	(b)	Fig. 10(f)				P^{tax}_{10000}	154,180	(c)	Fig. 6(c)				P^{tax}_{10000}	270,790
	A_{max}	400	c_b	805	P_{10000}	33,370		A_{max}	300	c_b	814	P_{10000}	36,280		A_{max}	300	c_b	1,138	P_{10000}	137,090

Fig. 30. Profits subject to tax calculation.

I learned about horse racing by chance. In January 2021, while I was visiting Obihiro (a remote city in Hokkaido) on a business trip, I stopped by the city's race course, which is a tourist attraction in the area. There I found a horse named Trumpet. Since playing trumpet is my hobby, I bet on the horse. Fortunately, with what must be beginner's luck, the horse came in first place. I became addicted to horse racing after that. Although I was just a beginner, I researched gambling and horse racing and found doubling to be a suspicious winning method. I tried doubling while making improvements to the method. I could make a few hundred yen, but at one point I incurred a loss of over 100,000 yen. I looked further into the historical data and started over. Once again, I made a few hundred yen, and I felt like the probability of a big loss had gone down a bit. Just then, I again incurred a loss of over 100,000 yen. At that moment, I realized how risky doubling can be. Using models based on about 1,000 past races leads to bankruptcy. I decided to create artificial race data based on simulations of tens of thousands of races before implementing a model. I resolved to make sure there were no pitfalls in the model. Even when good results were seen (e.g., A_{max}=5,900 with N4 of Type III in Table 33), I still didn't feel like putting the model into practice. The amount of data is small, and it could be just luck. Eventually, a deep valley will appear. I still had doubts. Yet, I continued going to the off-track betting shop every weekend. I purchased tickets for my favorite jockeys and for tickets related to my birthday for about 3,000 yen, limited to graded races. While doing this, I was looking for materials to build better models.

In gambling, there are good times and bad times. The bad times are long lasting, and the good times don't last long. Even if there was something that could lead to a method of winning, many scientists, including myself, haven't been paying enough attention. Like astrology, we do not consider it to be a good subject for research. Cardano's famous quotation also has an adverse effect. The forms of gambling that he targeted are mainly card games, dice, and roulette. It is said that the birthplace of modern horse racing was 16th-century England, and that it was introduced to Italy in the 18th century. It is not clear whether Cardano knew of the types of gambling such as with horse racing. However, times have evolved, and more horse racing data is now available. While I constructed the racing data in Section 2 using the assumption that the odds and the winner are decided by a roulette wheel, I often think that it is interesting that we assume the winner is decided by roulette wheels in actual horse races. We can construct roulette wheels from historical data including hit rates by rank, hit rates by horse number, win rates of jockeys, win rates by odds, and so on. If we observe the races from these viewpoints, we might be able to enjoy them more. I believe we have a chance to create solid methods for winning.

It is regrettable that there are some cases in 100,000 races where we have to prepare enough funds to cover 10,000 times the input amount. Even if we seem to be able to develop good methods through 10,000 races, there is a large risk when applied in 100,000 races. However, excluding some deep valleys, we can observe upward trending. I believe that there is no reason to give up.

Cardano's quotation has not yet been proven correct. According to a legend about horse racing, there are some people who use reliable methods of winning and who make large profits. I don't know the specific details or whether they have anything to do with doubling. Data analysis and model considerations related to horse racing are interesting, and sometimes I get so engrossed in my research that I can't sleep. At times like this, I recall Italian opera. Cardano's quotation will be overcome in the not-so-distant future. Then, with my favorite musicians, I will sing *Nessun Dorma* out loud like Luciano Pavarotti. "*Vincerò!*"

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